Imprecise Qualitative Spatial Reasoning

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Abstract

This paper addresses the issue of qualitative reasoning in imprecise spatial domains. In particular, the uncertainty in the nature of the spatial relationships between objects in space is represented by a set of possibilities. The approach to spatial reasoning proposed here is carried out in three steps. First, a transformation is carried out on the disjunctive set of possible relationships to derive their corresponding set of spatial constraints. Reasoning formulae are developed to propagate the set of identified constraints and finally a transformation is carried out on the resulting constraints to map them back to the domain of spatial relations to identify the result of the spatial composition. Two general equations form the basis for the propagation of the spatial constraints. A major advantage of this method is that reasoning with incomplete knowledge can be done by direct application of the reasoning formulae on the spatial objects considered, and thus eliminates the need for utilising the inordinate number of composition tables which must be built for specific object types and topology. The method is applied on spatial objects of arbitrary complexity and in a finite definite number of steps controlled by the complexity needed in the representation of objects and the granularity of the spatial relations required.

1 Introduction

Large spatial databases such as geographic databases are characterised by the need to store and manipulate substantial numbers of spatial objects and to provide effective and efficient means of retrieval and analysis. For example, a typical geographic database may contain hundreds of thousands of objects represented by polygons which are themselves represented by hundreds of points. Expensive computational geometry techniques as well as spatial data structures and indexing algorithms are normally employed in such databases. Spatial data in many applications of this sort are often imprecise or incomplete [9, 12]. This may be due in part to the inaccuracy of the measuring devices, or simply to the non-availability of the information. For example, a topographic data set may contain detailed representation of boundaries of large cities, but only representative centre points of smaller towns and villages. Hence, exact spatial relationships which these smaller objects may be involved in can't be precisely defined. It would therefore be useful, in these circumstances, for such systems to be able to encode this uncertainty in the representation of spatial objects and spatial relationships. More importantly, it would also be useful to reflect this imprecision in the manipulation and analysis of the data sets.

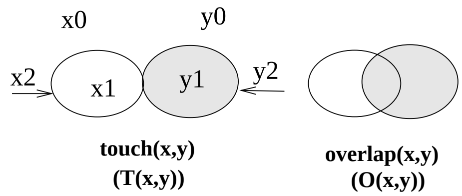
Qualitative Spatial Representation and Reasoning (QSRR) is an active field of AI research where formalisms for encoding and manipulating qualitative spatial knowledge are studied[3, 2]. The goal is for such techniques to complement and enhance the traditional methods, especially when precise information are neither available nor needed. A typical problem for qualitative reasoning techniques is the automatic composition of spatial relationships. For example, the derivation of the fact that a region x is not connected to another region z, using the knowledge that region x is inside a region y and y is either not connected to z or is touching z externally. Most approaches to QSRR are concerned with finding means of automating the composition of spatial relationships and ultimately the derivation of composition tables for different types of objects and relationships [6, 5, 4]. The problem is considered to be a major challenge to automatic theorem provers [1, 11].

This problem is complicated further when imprecise or incomplete knowledge is used as input to the composition process. In this case, knowledge is usually represented by a disjunctive set of relations, e.g. region x overlaps with or is inside region y. Every relation in the disjunctive set of relations need to be processed separately and the resulting sets of composed relations are intersected, or summed, to derive the final spatial composition result.

For example, if the relation between objects x and y is $disjoint \vee touch \vee overlap$ and y is $inside$ z, then the following four steps are needed to derive the composition between x and z.

1. $disjoint(x, y) \circ inside(y, z) \rightarrow \{disjoint \vee touch \vee overlap \vee inside\}$
2. $touch(x, y) \circ inside(y, z) \rightarrow \{meet \vee overlap \vee inside\}$
3. $overlap(x, y) \circ inside(y, z) \rightarrow \{overlap \vee inside\}$
4. The result is the sum of results of the above three steps, i.e. $\{disjoint \vee meet \vee overlap \vee inside\}$

In general, if the number of relations in the disjunctive sets for x, y and y, z are n and m respectively, then the number of spatial compositions is $n \times m$ and the total number of steps required to derive the result is $(n \times m) + 1$. The above method is dependent on the availability of composition tables between the objects involved and for the types of relations considered. Hence, composition tables must be either pre-computed or generated on the fly. What was also propagated in the above example is knowledge of possibilities about the spatial scene and not knowledge of facts. Few works have approached the problem of reasoning with imprecise spatial knowledge. In [8], Freksa used a semi-interval based temporal reasoning to deal with incomplete or imprecise knowledge. His method was based on capturing the relations between the starts and ends of intervals to represent a set of disjunctive relations. Eleven coarse relations were introduced and a diagrammatic representation of the disjunction was used. Hernandez [10] used a similar approach to define coarse relations between convex regions in the spatial domain. Both works are limited by the diagrammatic representations used and the fact that their methods are applicable only if the relations involved were conceptual neighbours [7]. Also, the method rely on looking the relations up in pre-computed composition tables as no general reasoning mechanism was proposed.



	y_0	y_1	y_2
x_0	1	1	1
x_1	1	0	0
x_2	1	0	1

(a)

	y_0	y_1	y_2
x_0	1	1	1
x_1	1	1	1
x_2	1	1	1

(b)

Figure 1: Different qualitative spatial relationships distinguished by identifying the appropriate intersection of components of the objects and the space, and their corresponding **intersection matrices** respectively.

In this paper, a new approach is proposed for reasoning with imprecise spatial knowledge. The approach is significant as it is carried out in only two steps, irrespective of the number of relations in the disjunctive sets noted above. Also, the method does not rely on the pre-computation of composition tables and hence may be applied to objects of arbitrary complexity.

The approach utilises the representation method proposed in [6] and inherits its generality in dealing with topological relations between objects of arbitrary complexity. The paper is organised as follows. In section 2, the approach used for the representation of topological relations is presented. The reasoning approach is then described in section 3. Several examples are given in section 4 to demonstrate the generality of the approach. In section 5, a discussion is given on how the method may be applied to representation and reasoning in the temporal domain. Conclusions and an outlook on future are given in section 6.

2 The Underlying Representation of Spatial Relations

Objects of interest and their embedding space are divided into components according to a required resolution. The method of representation of spatial relations has been proposed in earlier works [6] and is briefly reviewed here. Topological relations are represented through the intersection of objects components. The distinction of topological relations is dependent on the strategy used in the decomposition of the objects and their related spaces.

The complete set of spatial relationships are represented by combinatorial intersection of the components of one space with those of the other space.

If $R(x, y)$ is a relation of interest between object x and object y, and X and Y are the spaces associated with the objects respectively such that m is the number of components in X and l is the number of components in Y, then a spatial relation $R(x, y)$ can be represented by one state of the following equation:

$$
\begin{aligned}
R(x,y) &= X \cap Y \\
&= \left(\bigcup_{i=1}^{m} x_i\right) \cap \left(\bigcup_{j=1}^{l} y_j\right) \\
&= (x_1 \cap y_1, \cdots, x_1 \cap y_m, x_2 \cap y_1, \cdots, x_n \cap y_m)
\end{aligned}
$$

The intersection $x_i \cap y_j$ can be an empty or a non-empty intersection. The above set of intersections shall be represented by an intersection matrix, as follows,

$R(x, y) =$

	y_0	y_1	y_2	$\cdots$
x_0				
x_1				
x_2				
$\vdots$				

For example, spatial relationships and their corresponding intersection matrices are shown in figure 1. The component x_2 has an empty intersection with y_1 in 1(a) and a non-empty intersection in 1(b).

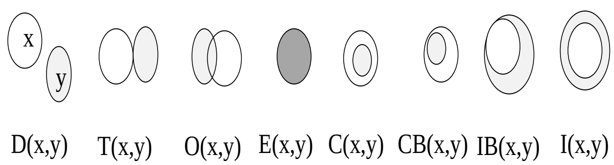
2.1 Mapping Component Intersections into Relations

The intersection matrix is in fact a set of intersection constraints whose values identifies specific spatial relationships. Figure 2 represents the mapping between intersection constraints and set of spatial relations in the case of two convex areal objects. Table entries represent the resulting set of possible relations if the result of the intersection of the corresponding components is non-empty (or (1)). For example, if $x_0 \cap y_2 = 1$ is the only intersection known then the relationship between objects x and y is $D \vee T \vee O \vee I \vee IB$, and so on. If the result of the intersection is the empty set (or (0)), then the possible set of relations will be the complement of the sets shown. If $x_0 \cap y_2 = 0$, then the corresponding table entry will be $E \vee C \vee CB$. The possible relations between objects x and y can therefore be derived from the combination (set intersection) of all the table entries. An example is given in figure 3 where in 3(a) the intersection matrix for objects x and y is shown (with unknown value for $x_2 \cap y_2$). In 3(b) the mapping of the intersections into possible relations is given using the table (and its complement) in figure 2(b). The spatial relations between objects x and y is then derived from the set intersection of all table entries to be $I(x, y) \vee IB(x, y)$ shown in 3(c).

3 The Approach

The imprecise knowledge in the form of a disjunctive set of spatial relations is to be represented as a set of intersection constraints. For example, the set of relations (T, O, E, CB, IB) shown in figure 4(a) can be expressed precisely by one constraint only, namely, $x_2 \cap y_2 = 1$, as shown in 4(b).

The process of spatial reasoning can be defined as the process of propagating the intersection constraints of two spatial relations (for example, $R_1(A, B)$ and



(a)

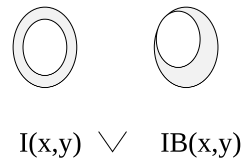
	y_0	y_1	y_2
x_0	ALL	$(D \vee T \vee O \vee I \vee IB)$	$(D \vee T \vee O \vee I \vee IB)$
x_1	$(D \vee T \vee O \vee C \vee CB)$	$(O \vee I \vee IB \vee C \vee CB \vee E)$	$(O \vee C \vee CB)$
x_2	$(D \vee T \vee O \vee C \vee CB)$	$(I \vee IB \vee O)$	$(T \vee O \vee IB \vee CB \vee E)$

(b)

Figure 2: (a) Set of possible relations between two convex areal objects. (b) Mapping the non-empty intersection of object components into a disjunctive set of possible relations.

	y_0	y_1	y_2
x_0	1	1	1
x_1	0	1	0
x_2	0	1	$0 \vee 1$

(a)

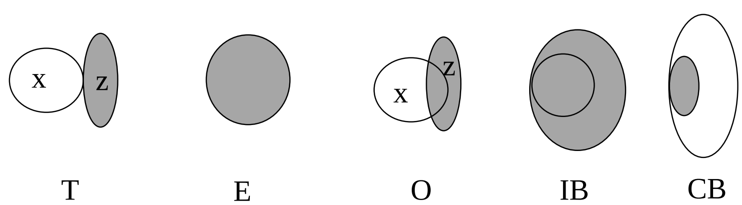


(c)

	y_0	y_1	y_2
x_0	ALL	$(D \vee T \vee O \vee I \vee IB)$	$(D \vee T \vee O \vee I \vee IB)$
x_1	$(I \vee IB \vee E)$	$(O \vee I \vee IB \vee C \vee CB \vee E)$	$(I \vee IB \vee O \vee T \vee E)$
x_2	$(I \vee IB \vee E)$	$(I \vee IB \vee O)$	$(T \vee O \vee IB \vee CB \vee E)$ OR $(D \vee I \vee C)$

(b)

Figure 3: (a) An example intersection matrix with unknown value for $x_2 \cap y_2$. (b) Mapping intersections of individual components to possible relations. (c) Possible relations between objects as the common subset in all the table entries.



(a)

	y_0	y_1	y_2
x_0	?	?	?
x_1	?	?	?
x_2	?	?	1

(b)

Figure 4: (a) Set of spatial possible relations between two spatial objects. (b) Their representation by one constraint $x_2 \cap y_2 = 1$.

$R_2(B,C)$), to derive a new set of intersections between objects. The derived constraints can then be mapped to a specific spatial relation (i.e. the relation $R_3(A,C)$).

Let $X = \bigcup_{i=1}^{m} x_i$ and $Z = \bigcup_{k=1}^{n} z_k$ represent the spaces X and Z associated with objects x and z respectively. m and n are the total number of components in those spaces. If $y_j \subset Y$ and Y is the embedding space for the common object in the composition relation and since $X = Y = Z$, it follows that $(X \supset y_j) \wedge (y_j \subset Z)$.

In general the intersection of two components can take one of three values: 0 or 1 or ? where 0 indicates an empty intersection, 1 a non-empty intersection and ? indicates either a 0 or a 1. (unknown value). Hence, if $P = \{0, 1, ?\}$ then $y_j \subset X \rightarrow \forall x_i \in X(y_j \cap x_i = P_q)$ where $P_q \in P$. Similarly, $y_j \subset Z \rightarrow \forall z_k \in Z(y_j \cap z_k = P_q)$

The reasoning process can be carried in two steps, namely:

1. **Multiplication Operation:** on the intersection relations between every component from the intermediate space and every component of the other two spaces.

2. **Addition Operation:** on the results of multiplication for all the components of the intermediate space.

The multiplication operation can be expressed as follows:

$$(X \cap Z)_{y_j} = \left(\bigcup_{i=1}^{m} x_i \cap y_j \right) \star \left(\bigcup_{k=1}^{n} z_k \cap y_j \right) \tag{1}$$

where $X \cap Z)_{y_j}$ are the propagated intersection relations between components of the spaces X and Y based on their intersection with the component y_j of space Y. The addition operation can be expressed as follows:

$$(X \cap Z) = \Sigma_{j=1}^{l} (X \cap Z)_{y_j} \tag{2}$$

where l is the total number of components of space Y.

Substituting 1 in 2 we get the general reasoning formula below.

General Reasoning Formula

$$X \cap Z = \Sigma_{j=1}^{l} \left((\bigcup_{i=1}^{m} x_i \cap y_j) \star (\bigcup_{k=1}^{n} z_k \cap y_j) \right) \tag{3}$$

Note that there is no restriction on the application of formula 3 to a single or a set of components of the intermediate space, hence a general form for formula 1 can be stated as follows.

$$(X \cap Z)_{y'} = \left(\bigcup_{i=1}^{m} x_i \cap y' \right) \star \left(\bigcup_{k=1}^{n} z_k \cap y' \right) \tag{4}$$

where $y' \subseteq Y$, for example $y' = y_1 \cup y_2$..

The two formulae 3 and 4, constitute the general spatial reasoning method for incomplete or uncertain knowledge. Formula 3 need to be applied in all

$\star$	0	?	1	$?^+$	1^+
0	0	0	0	0	0
?	0	?	?	?	?
1	0	?	1	?	1
$?^+$	0	?	?	?	$?_\alpha$
1^+	0	?	1	?	$?_\alpha$

Table 1: Multiplication Table for incomplete knowledge.

$+$	0	?	1	$?_\beta$
0	0	?	1	$?_\beta$
?	?	?	1	$?_\beta$
1	1	1	1	1
$?_\alpha$	$?_\alpha$	$?_\alpha$	1	$?_\alpha \wedge ?_\beta$

Table 2: Addition Table for incomplete knowledge.

cases, while 4 is only needed to be applied whenever a constraint exist of the following form:

$$x_i \cap y_j \;=\; ? \;\wedge\; x_i \cap y_{j+1} \;=\; ? \;\wedge\; x_i \cap (y_j \cup y_{j+1}) = 1 \tag{5}$$

i.e. the intersection of x_i with both y_j and y_{j+1} cannot be empty. Then using $y' = (y_j \cup y_{j+1})$ will give $x_i \cap y' = 1$.

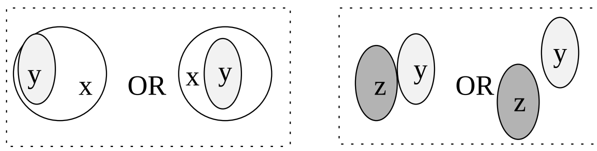
To distinguish the constraint in 5 from a non-related constraint of the form: $x_i \cap y_j \;=\; ? \;\wedge\; x_i \cap y_{j+1} \;=\; ? \;\wedge\; x_i \cap (y_j \cup y_{j+1}) =?$, a label will be used and equation 5 can be rewritten as: $x_i \cap y_j \;=\; ?_a \;\wedge\; x_i \cap y_{j+1} \;=\; ?_a$ The added letter a indicates that the two constraints are related.

A constraint of the type 5 can be used as either an input to the reasoning task or an output of it. If the number of components of space x that has non-empty intersections with y_j is > 1, then this case is distinguished with a lable 1^+ instead of 1. Similarly, if the number of components of space x that has non-defined intersections (?) with y_j is > 1 then this case is distinguished with a $?^+$ instead of ?.

Accordingly the multiplication and addition tables of our method are as shown in figure 1 and 2.

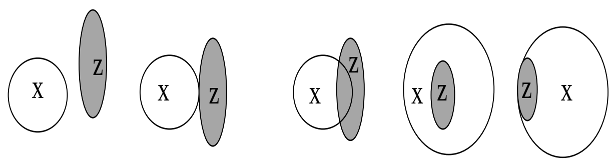
Note that in the addition table, we used $?_\alpha$ and $?_\beta$ since we add results for different components of space Y, i.e. $?_\alpha + ?_\beta = ?_\alpha \wedge ?_\beta$.

Equations 3 and 4 and the multiplication and addition tables represent the general space algebra for reasoning with incomplete or uncertain knowledge. The algebra makes no restriction on the complexity of the objects used or the completeness or uncertainty of the knowledge of the topological relations involved.



x_2	x_1	x_0		z_0	z_1	z_2
1	1	1	y_0	1	1	1
0	1	0	y_1	1	0	0
?	1	0	y_2	1	0	?

(a)



(b)

Figure 5: (a) Example reasoning problem with incomplete knowledge. (b) Resulting possible set of relations between x and z.

4 Examples of Spatial Reasoning with Incomplete Knowledge

4.1 Example 1

Consider the reasoning problem where the relations between simple convex areal objects x, y and z are: $C(x, y)$ *or* $CB(x, y)$ between x and y and $D(y, z)$ *or* $T(y, z)$ between y and z, as shown in figure 5(a). This indefiniteness is reflected in the intersection matrices in the figure where $x_2 \cap y_2 =?$ and $y_2 \cap z_2 =?$.

It is required to derive the possible relationships between objects x and z.

Applying the general topological reasoning equation 3 on y_0, y_1 and y_2 using the multiplication table, we get the following,

- y_0 intersections: ($n'' = n$ and $m'' = m$ - second general constraint)

$$(X \cap Z)_{y_0} \quad \rightarrow \quad \{x_0, x_1, x_2\} \cap \{z_0, z_1, z_2\} =?$$

- y_1 intersections: $x_1 \cap z_0 = 1$ and $(X - x_2) \cap (Z - z_0) = \phi$.

- y_2 intersections: ($n'' > 0 \wedge n' > 1 \wedge m'' > 0 \wedge m' > 1$, i.e. a constraint of the type described in section **??**(**??**) above.

  $x_1 \cap z_0 =?_a$
  $x_1 \cap z_2 =?_a$
  $z_0 \cap x_1 =?_b$
  $z_0 \cap x_2 =?_b$
  $x_0 \cap (z_0 \cup z_1 \cup z_2) = 0$

$x_2 \cap z_2 =?$
$z_1 \cap (x_0 \cup x_1 \cup x_2) = 0$

Using the addition table we get the following:

	y_0	y_1	y_2	$\Sigma_{j=1}^{3} y_j$
$x_0 \cap z_0$	?	0	0	?
$x_0 \cap z_1$	?	0	0	?
$x_0 \cap z_2$	?	0	0	?
$x_1 \cap z_0$	?	1	$?_a \wedge ?_b$	1
$x_1 \cap z_1$	?	0	0	?
$x_1 \cap z_2$	?	0	$?_a$	$?_a$
$x_2 \cap z_0$	?	0	$?_b$	$?_b$
$x_2 \cap z_1$	?	0	0	?
$x_2 \cap z_2$	?	0	?	?

Also from the first general constraint $x_0 \cap z_0 = 1$. Compiling the above intersection we get the following resulting intersection matrix,

	z_0	z_1	z_2
x_0	1	?	?
x_1	1	?	?
x_2	?	?	?

Mapping the above matrix into spatial relations, as shown earlier in table 2(b), gives the disjunctive set: $R(X, Z) = D \vee T \vee O \vee C \vee CB$ as shown in figure 5(b).

4.2 Example 2

The resulting relationships in the above example gave the intersection of x_2 as $x_2 \cap \{z_0, z_1, z_2\} =?$, i.e. no constraints are propagated for the component x_2. If a new fact is added such that $x_2 \cap \{z_0, z_2\} = 1$, i.e. $x_2 \cap z_0 =?_a$ and $x_2 \cap z_2 =?_a$ and in this case the relationships between objects x and z are to be further composed with a relationship $C(z, q)$ between objects z and q as shown in figure 6.

Applying the general topological reasoning equation 3 on z_0, z_1 and z_2 and using the multiplication table, we get the following,

- z_0 intersections:

$$\begin{aligned}(X \cap Q)_{z_0} = \quad & \rightarrow \quad (x_0 \cap q_0 = 1) \wedge (x_1 \cap q_0 = 1) \\ & \wedge \quad (x_2 \cap q_0 =?) \\ & \wedge \quad \{q_1, q_2\} \cap \{x_0, x_1, x_2\} = \phi\end{aligned}$$

- z_1 intersections:

$$\{q_0, q_1, q_2\} \cap \{x_0, x_1, x_2\} =?$$

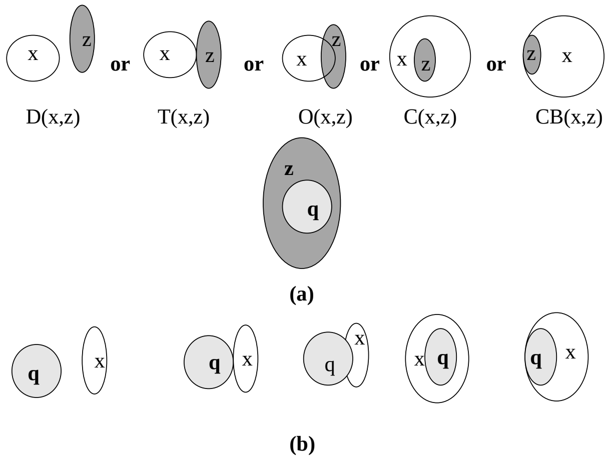


Figure 6: (a) Further composition of the spatial relations between x and z from the previous example with the relation $C(z, q)$ to get the possible relations between x and q in (b).

- z_2 intersections:

$$(q_0 \cap \{x_0, x_1, x_2\} = ?) \wedge \{q_1, q_2\} \cap \{x_0, x_1, x_2\} = \phi$$

From the previous example we have that $x_2 \cap z_0 = ?_a$ and $x_2 \cap z_2 = ?_a$.

Applying equation 4 for x_2 only, we get the following,

- x_2 intersections:

$$x_2 \cap (z_0 \cup z_2) = 1 \wedge (z_0 \cup z_1) \cap q_0 = 1 \rightarrow x_2 \cap q_0 = 1$$

Compiling the above intersection we get the following resulting intersection matrix,

	q_0	q_1	q_2
x_0	1	?	?
x_1	1	?	?
x_2	?	?	?

Mapping the above matrix into spatial relations, as shown earlier in table 2(b), gives the disjunctive set: $R(X, Q) = D \vee T \vee O \vee C \vee CB$ as shown in figure 6(b).

4.3 Example 3: Composition with indefinite and related constraints

Consider the relations between objects x and y and z as shown in figure 7 ($R(x, y) = T(x, y) \vee C(x, y) \vee CB(x, y)$ and $R(y, z) = IB(y, z)$). Their representative intersection matrices are as follows:

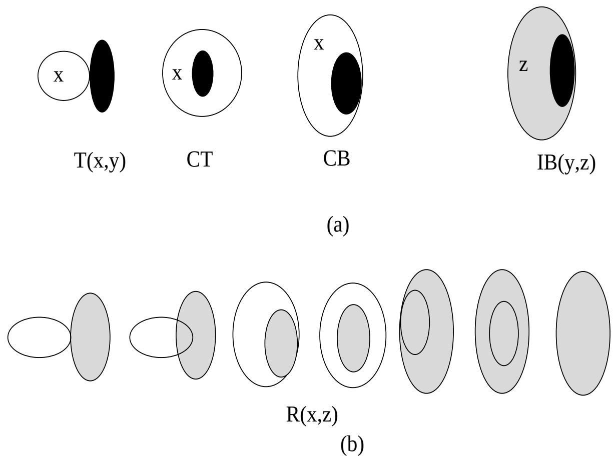


Figure 7: (a) Composition with indefinite and related constraints. (b) The composition result.

x_2	x_1	x_0		z_0	z_1	z_2
1	1	1	y_0	1	1	1
0	?	?	y_1	0	1	0
$?_a$	$?_a$	?	y_2	0	1	1

Applying the general formula 3 on y_0, y_1 and y_2 and using the multiplication table, we get the following,

- y_0 intersections:

$$\{x_0, x_1, x_2\} \cap \{z_0, z_1, z_2\} = 1$$

- y_1 intersections:

$$x_1 \cap z_1 =? \wedge x_0 \cap z_1 =? \wedge x_2 \cap z_1 =?$$

- y_2 intersections:

$$\begin{aligned} x_1 \cap z_1 &=?_a \quad \wedge \quad x_1 \cap z_2 =?_a \\ x_2 \cap z_1 &=?_a \quad \wedge \quad x_2 \cap z_2 =?_a \\ x_0 \cap z_1 &=? \quad \wedge \quad x_0 \cap z_2 =? \end{aligned}$$

Also from the first general constraint $x_0 \cap z_0 = 1$. Compiling the above intersection we get the following resulting intersection matrix,

	z_0	z_1	z_2
x_0	1	?	?
x_1	?	$?_a$	$?_a$
x_2	?	$?_a$	$?_a$

I.e. $\{x_1, x_2\} \cap \{z_1, z_2\} = 1$. Mapping the constraint back will result in the exclusion of the disjoint relation only, and hence, $R(X, Z) = T \vee O \vee E \vee I \vee IB \vee C \vee CB$.

5 Conclusions

A general approach to spatial reasoning over imprecise topological relations is proposed. The approach is applicable to objects with random complexity. The method builds on and genarlises previous work in [6] where spatial relations are represented by the intersection of object and space components. Spatial reasoning is carried out in three steps. First a transformation is used to map the imprecise input relations into a specific set of known constraints. Spatial reasoning is carried out on the constraints to derive a resulting set of constraints and finally the resulting constraints are mapped back into a set of possible relations between the objects considered. The method eliminates the need for the development and utilisation of composition tables in the spatial reasoning process. It has also been briefly shown how to adapt the method for representation and reasoning in the temporal domain. The homogeneous treatment of space a time is a subject of much research and shall be investigated further in future works.

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