Order in Space : A General Formalism for Spatial Reasoning

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Abstract

In this paper we propose a general approach for reasoning in space. The approach is composed of a set of two general constraints to govern the spatial relationships between objects in space, and two rules to propagate relationships between those objects. The reasoning method is applicable to objects of random complexity and in any space dimension. The approach is based on a uniform representation of the topology of the space as a connected set of components and the representation of the relationships between those objects by the intersection of the space components. Several examples are used to illustrate the generality of the proposed method. The approach is also shown to be applicable to reasoning in the temporal domain and is used to explain some phenomenon related to the reasoning process, namely, conceptual neighbourhood and definite and indefinite compositions. A major advantage of the method is that reasoning between objects of any complexity can be achieved in a definite limited number of steps. Hence, the incorporation of spatial reasoning mechanisms in spatial information systems becomes plausible.

1 Introduction

A new generation of information systems tailored to handling spatial knowledge has been actively evolving over the past few years. The representation of qualitative spatial knowledge and reasoning about it are central tasks in many such systems for example, in CAD/CAM applications [6], image processing [21], geographic information systems [27], robotics, qualitative dynamics [16, 25]. Qualitative spatial models are also used in non-spatial domains, for example, in natural language understanding in software engineering in the visulisation and animation of programs [22]. In all these applications, qualitative as opposed to quantitative treatment of spatial knowledge is needed. Precise information required in quantitative methods are usually neither available nor needed.

While the qualitative treatment of the temporal knowledge is an established research area where different approaches exist for the representation of temporal entities (intervals, point, moments), their relations (interval and point algebra) and reasoning over them (composition tables and constraint networks), a general treatment of spatial knowledge is still lacking []. Spatial reasoning is considered as a challenge to automatic theorem provers [28]. Composition tables need to be built for every new type of objects considered and techniques to derive them automatically presents a challenge. Phenomena such as conceptual neighbourhood needs explanation [8].

General formalisms for the representation and reasoning of qualitative spatial knowledge are needed to provide unambiguous definitions of spatial relations to facilitate their automated processing. Some research work [20] tried to exploit the well developed treatment of temporal knowledge [5] in handling the representation in the spatial domain. However, the multi-dimensionality and complexity of the topology of spatial entities, as opposed to the uni-dimensionality of temporal entities and their simpler topology prevented the generality of these approaches. Other approaches also exist for example those by Cohn et al [11] and by Egenhofer [13]. However, general approaches which handles reasoning over objects of different types and random complexity are not yet achieved.

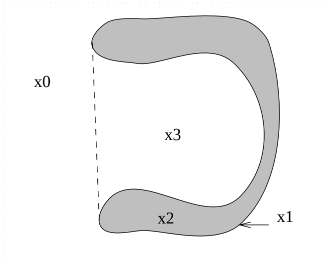
In this paper a general reasoning formalism for qualitative spatial relations is proposed which is composed of a set of general constraints and set of general reasoning rules. Both the rules and the constraints are based on the a uniform representation of the topology of the objects, their embedding space and the relationships between them. The rest of the paper is structured as follows. Section 2 describes the proposed approach by describing the underlying representation methodology and the reasoning formalism. Examples are given to show how the approach can be used to represent and reason over relationships between objects with random complexity. In section 3, the reasoning approach is utilized to analyze and discuss central issues of qualitative reasoning. In particular, it is shown under which conditions the composition of spatial relationships is definite or in-definite. A possible explanation on how the phenomenon of conceptual neighbourhood occurs is discussed and finally the application of the same approach to the representation and reasoning in the temporal domain is given. Section 5 gives a comparative description of related approaches and some conclusions and a view over future work are given in section 6.

2 The Approach

The approach proposed in this paper for reasoning in space is composed of two parts: a) general constraints to govern the spatial relationships between objects in space, and b) general rules to propagate relationships between objects in space. Both the constraints and the rules are based on a uniform representation of the topology of the objects and the representation of the relationships between those objects. The representation methodology is first described and examples are used to demonstrate how relationships between objects of random complexity can be represented.

2.1 The Underlying Representation

The method used for representing the space and the objects is similar to the space vocabulary described in [15] where objects of interest and their embedding space are divided intro components according to a required resolution. The connectivity of those components is explicitly represented. The representation of the spatial relations uses a similar approach to that used in [13] where relationships are described by the intersection of the objects components [2, 3].



(a)

	x_0	x_1	x_2	x_3
x_0	-	1	0	1
x_1	1	-	1	1
x_2	0	1	-	0
x_3	1	1	0	-

(b)

x_0			
1	x_1		
0	1	x_2	
1	1	0	x_3

(c)

Figure 1: (a) Possible decomposition of a concave-shaped object and its embedding space. (b) adjacency matrix corresponding to the shape in (a). 1 in the matrix represents that the corresponding elements are connected and a 0 represents non-connection. (c) Half the symmetric connectivity matrix in (b) is sufficient to capture object representation.

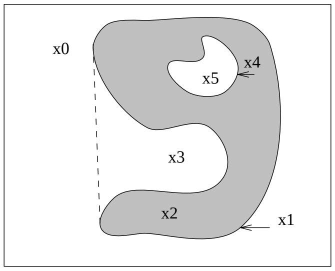
2.1.1 The Underlying Representation of Object Topology

In this section an approach for the representation of the topology of objects with arbitrary complexity is described. Let S be the space in which the object is embedded. The object and its embedding space are assumed to be *dense* and *connected*. The embedding space is also assumed to be infinite. The object and its embedding space are decomposed into components which reflects the objects and space topology such that,

1. No overlap exist between any of the representative components.
2. The union of the components is equal to the embedding space.

The topology of the object and the embedding space can then be described by a matrix whose elements represent the connectivity relations between its components. This matrix shall be denoted *adjacency matrix*. In figure 1(a) a possible decomposition of a concave shaped object (for example an island with a bay) and its embedding space is shown and in 1(b) the adjacency matrix for its components is presented. The object is represented by two components a linear component x_1 (the shore line of the island) and an areal component x_2 and the rest of its embedding space is represented by a finite areal component x_3 (representing the bay of the island) and infinite areal component x_0 representing the surrounding area. The fact that two components are connected is represented by a (1) in the adjacency matrix and by a (0) otherwise. Since connectivity is a symmetric relation, the resulting matrix will be symmetric around the diagonal. Hence, only half the matrix is sufficient for the representation of the object's topology and the matrix can be collapsed to the structure in figure 1(c). In the decomposition strategy, the complement of the object in question shall be considered to be infinite. The suffix 0 (x_0) is used to represent this component.

Note that different decomposition strategies for the objects and their embedding spaces can be used according to the precision of the relations required and the specific application considered. The higher the resolution used (or the finer the components of the space and the objects), the higher the precision of the resulting set of relations in the domain considered. For example, if we were interested in studying

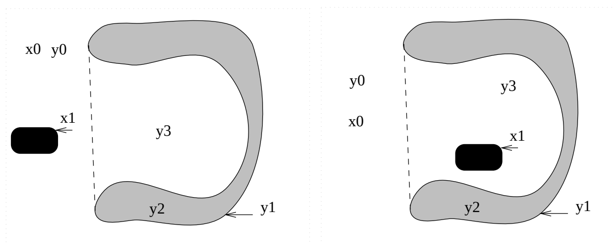


(a)

x_0					
1	x_1				
0	1	x_2			
1	1	0	x_3		
0	0	1	0	x_4	
0	0	0	0	1	x_5

(b)

Figure 2: Different decomposition of the concave-shaped island in figure to capture more details and correspondingly increase the resolution of the objects represented. x_4 and x_5 represent a lake on the island.



(a) (b)

Figure 3: Different qualitative spatial relationships can be distinguished by identifying the appropriate components of the objects and the space.

the lakes on the island in figure 1(a), more components to represent the lakes have to be used as in figure 2.

2.1.2 The Underlying Representation of Spatial Relations

In this section, the representation of the topological relations through the intersection of their components [13] is adopted and generalized for objects of arbitrary complexity.

Distinction of topological relations is dependent on the strategy used in the decomposition of the objects and their related spaces. For example, in figure 3 different relationships between two objects representing a ship (x) and an island (y) are shown, where in 3(a) the ship is outside the bay and in 3(b) the ship is inside the bay. The concave region representing the island (y) is decomposed into two components x_1 and x_2 and the rest of the space associated with x is decomposed into two components (x_3 representing the bay and x_0 representing the rest of the ocean). Note that the component x_3 is a virtual component, i.e. with no physical boundary to delineate its spatial extension. It is the identification of this component that makes the distinction between the two relationships in the figure.

The complete set of spatial relationships are represented by combinatorial intersection of the components

	y_1	y_2	y_3	y_0
x_1	0	0	0	1
x_2	0	0	0	1
x_0	1	1	1	1

(a)

	y_1	y_2	y_3	y_0
x_1	0	0	1	0
x_2	0	0	1	0
x_0	1	1	1	1

(b)

Figure 4: The corresponding **intersection matrices** for the relationships in figure 3 respectively. Each element in the matrix represent the result of the intersection of the corresponding elements, (1) for a non-empty intersection and (0) for an empty intersection.

of one space with those of the other space.

If $R(x, y)$ is a relation of interest between object x and object y, and X and Y are the spaces associated with the objects respectively such that n is the number of components in X and m is the number of components in Y, then a spatial relation $R(x, y)$ can be represented by one state of the following equation:

$$\begin{aligned} R(x,y) &= X \cap Y \\ &= \left(\bigcup_{i=1}^{n} x_i \right) \cap \left(\bigcup_{j=1}^{m} y_j \right) \\ &= (x_1 \cap y_1, x_1 \cap y_2, \cdots, x_1 \cap y_m, x_2 \cap y_1, \cdots, x_n \cap y_m) \end{aligned}$$

The intersection $x_i \cap y_j$ can be an empty or a non-empty intersection. The above set of intersections shall be represented by an intersection matrix, similar to that used in [14] as follows,

$R(x, y) =$

	y_0	y_1	y_2	$\cdots$
x_0				
x_1				
x_2				
$\vdots$				

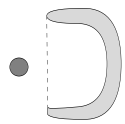
For example, the intersection matrices corresponding to the spatial relationships in figure 3 are shown in figure 4. The components x_1 and x_2 have a non-empty intersection with y_0 in 4(a) and with y_3 in 4(b).

Different combinations in the intersection matrix can represent different qualitative relations. The set of sound spatial relationships between objects is dependent on the particular domain studied. For example, in considering relationships between two line objects in a network analysis application we might be interested in only those relationships where end points of lines are in contact. Also, properties of the objects would affect the set of possible spatial relationships that can exist between them. For example, if one object is solid object and the other is permeable, there cannot be any intersection of the inside of the solid object with any other component of the other object. Also, objects of different size or shape cannot be involved in certain spatial relations such as `equal` or `contain` between the smaller and the larger object.

The example in figure 5 demonstrates the spatial relations that can exist between two *solid* objects, one having the shape of a convex region and the other a concave one. The example can be used to represent

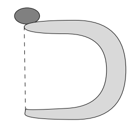
$R_(1) =$

	y_1	y_2	y_3	y_0
x_1	0	0	0	1
x_2	0	0	0	1
x_0	1	1	1	1



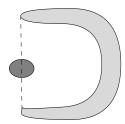
$R_2 =$

	y_1	y_2	y_3	y_0
x_1	1	0	0	1
x_2	0	0	0	1
x_0	1	1	1	1



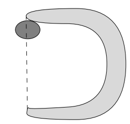
$R_3 =$

	y_1	y_2	y_3	y_0
x_1	0	0	1	1
x_2	0	0	1	1
x_0	1	1	1	1



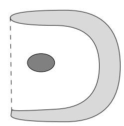
$R_4 =$

	y_1	y_2	y_3	y_0
x_1	1	0	1	1
x_2	0	0	1	1
x_0	1	1	1	1



$R_5 =$

	y_1	y_2	y_3	y_0
x_1	0	0	1	0
x_2	0	0	1	0
x_0	1	1	1	1



$R_6 =$

	y_1	y_2	y_3	y_0
x_1	1	0	1	0
x_2	0	0	1	0
x_0	1	1	1	1

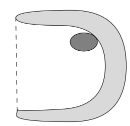


Figure 5: A set of 6 spatial relationships between two solid bodies simulating an example of a ball falling into a container full of liquid. The container along with the liquid inside it constitute one object y and the ball is object x. The decomposition of objects are as in figure 3.

many situations, for example, a solid object falling into a container full of liquid, a ball thrown into a net, or a ship entering a bay of an island, etc. In the figure the six possible spatial relationships that can occur between the two solid objects are shown along with their intersection matrices.

2.2 The General Reasoning Formalism

The reasoning approach is composed of : a) general constraints to govern the spatial relationships between objects in space, and b) general rules to propagate relationships between objects in space.

2.2.1 General Constraints

The intersection matrix is in fact a set of constraints whose values identifies specific spatial relationships. For example, the constraints used to represent the relationship in figure 3(a) are $x_1 \cap y_1 = 0, x_1 \cap y_2 = 0, x_1 \cap y_3 = 0, x_1 \cap y_4 = 1, \cdots$

The process of spatial reasoning can be defined as the process of propagating the constraints of two spatial relations (for examples, $R_1(A,B)$ and $R_2(B,C)$), to derive a new set of constraints between objects. The derived constraints can then be mapped to a specific spatial relation (i.e. the relation $R_3(A,C)$).

A subset of the set of constraints defining all spatial relation are general and are applicable to any relationship between any objects. These general constraints are a consequence of the initial assumptions used in the definition of the object and space topology. The identification of these constraints is useful and can be used in checking the correctness of the relations and shall be used later in the paper to give some insight in the propagation of spatial relations.

The two general constraints are:

1. Every unbounded (infinite) component of one space must intersect with at least one unbounded (infinite) component of the other space.

   Intuitively this rule says that it is impossible for an infinite component in the space to only have an intersection with finite component(s). In this case the infinite component becomes a subset of the finite component(s) which is not possible. In figure 5, x_0 and y_0 always have a non-empty intersection.

2. Every component from one space must intersect with at least one component from the other space.

   If one component of one space does not intersect with any component of the other space, either the two spaces are not equal or the spaces are not *connected*. Both conditions are excluded by the initial assumptions. This implies that there cannot exist a row or a column in the intersection matrix whose elements are all empty intersections, hence the combinatorial cases in the matrix where this case exists can be ignored.

2.2.2 General Reasoning Rules over Qualitative Spatial Relationships

In this section two general reasoning rules for the propagation of intersection constraints are presented which govern the composition of spatial relations. Composition of spatial relations is the process through which the possible relationship(s) between two object x and z is derived given two relationships: R_1 between x and y and R_2 between y and z. The rules are characterized by the ability to reason over spatial relationships between objects of arbitrary complexity in any space dimension. These rules allow for the automatic derivation of the composition (transitivity) tables between any spatial shapes [1]- a task considered to be a challenge to automatic theorem provers [28].

Reasoning Rules

Composition of spatial relations using the *intersection-based* representation approach is based on the transitive property of the subset relations. In what follows the following subset notation is used. If x' is a set of components (set of point-sets) $\{x_1, \cdots, x_n\}$ in a space X, and y_j is a component in space Y, then $\sqsubseteq$ denotes the following subset relationship.

- $y_j \sqsubseteq x'$ denotes the subset relationship such that: $\forall x_i \in x'(y_j \cap x_i \neq \phi) \wedge y_j \cap (X - x_1 - x_2 \cdots - x_n) = \phi\ where\ i = 1, \cdots n$. Intuitively, this symbol indicates that the component y_j intersects with every set in the set x' and does not intersect with any set outwith x'.

If x_i, y_j and z_k are components of objects x, y and z respectively, then if there is a non-empty intersection between x_i and y_j, and y_j is a subset of z_k, then it can be concluded that there is also a non-empty intersection between y_j and z_k.

$$(x_i \cap y_j \neq \phi)\ \wedge\ (y_j \subseteq z_k) \rightarrow (x_i \cap z_k \neq \phi)$$

This relation can be generalized in the following two rules. The rules describe the propagation of intersections between the components of objects and their related spaces involved in the spatial composition.

Rule 1: Propagation of Non-Empty Intersections

Let $x' = \{x_1, x_2, \cdots, x_{m'}\}$ be a subset of the set of components of space X whose total number of components is m and $m' \leq m$; $x' \subseteq X$. Let $z' = \{z_1, z_2, \cdots, z_{n'}\}$ be a subset of the set of components of space Z whose total number of components is n and $n' \leq n$; $z' \subseteq Z$. If y_j is a component of space Y, the following is a governing rule of interaction for the three spaces X, Y and Z.

$$
\begin{aligned}
(x' \sqsupseteq y_j) \ \wedge \ (y_j \sqsubseteq z') \quad &\rightarrow \quad (x' \cap z' \neq \phi) \\
&\equiv \quad (x_1 \cap z_1 \neq \phi \vee x_1 \cap z_2 \neq \phi \vee \cdots \vee x_1 \cap z_{n'} \neq \phi) \\
&\qquad \wedge(x_2 \cap z_1 \neq \phi \vee x_2 \cap z_2 \neq \phi \vee \cdots \vee x_2 \cap z_{n'} \neq \phi) \\
&\qquad \wedge \cdots \\
&\qquad \wedge(x_{m'} \cap z_1 \neq \phi \vee x_{m'} \cap z_2 \neq \phi \vee \cdots \vee x_{m'} \cap z_{n'} \neq \phi)
\end{aligned}
$$

The above rule states that if the component y_j in space Y has a positive intersection with every component from the sets x' and z', then each component of the set x' must intersect with at least one component of the set z'.

The constraint $x_i \cap z_1 \neq \phi \ \vee \ x_i \cap z_2 \neq \phi \cdots \ \vee \ x_i \cap z_{n'} \neq \phi$ can be expressed in the intersection matrix by a label, for example the label a in the following matrix indicates $x_1 \cap (z_2 \cup z_4) \neq \phi$ (x_1 has a positive intersection with z_2, or with z_4 or with both). A $-$ in the matrix indicates that the intersection is either positive or negative.

	z_1	z_2	z_3	z_4	$\cdots$	z_n
x_1	$-$	a	$-$	a	$-$	$-$

If the process of propagating the intersections produced two constraints which overlap, for example,

$$x_1 \cap z_1 \neq \phi \ \vee \ x_1 \cap z_2 \neq \phi \ \vee \ x_1 \cap z_3 \neq \phi \tag{1}$$

$$x_1 \cap z_1 \neq \phi \ \vee \ x_1 \cap z_3 \neq \phi \tag{2}$$

only the stronger one, ((2) in this case) has to be represented. Similarly, if the constraint $x_1 \cap z_1 \neq \phi$ is derived, then both constraints 1 and 2 need not be represented. However, they shall be replace by a - indicating non-definite intersection.

Rule 1 represents the propagation of non-empty intersections of components in space. A different version of the rule for the propagation of empty intersections can be stated as follows.

Rule 2: Propagation of Empty Intersections

Let $z' = \{z_1, z_2, \cdots, z_{n'}\}$ be a subset of the set of components of space Z whose total number of components is n and $n' < n$; $z' \subset Z$. Let $y' = \{y_1, y_2, \cdots, y_{l'}\}$ be a subset of the set of components of space Y whose total number of components is m and $l' < l$; $y' \subset Y$. Let x_i be a component of the space X. Then the following is a governing rule for the spaces X, Y and Z.

$$(x_i \sqsubseteq y') \ \wedge \ (y' \sqsubseteq z') \quad \rightarrow \quad (x_i \cap (Z - z_1 - z_2 \cdots - z_{n'}) = \phi)$$

Remark: if $n' = n$, i.e. x_i may intersect with every element in Z, then no empty intersections can be

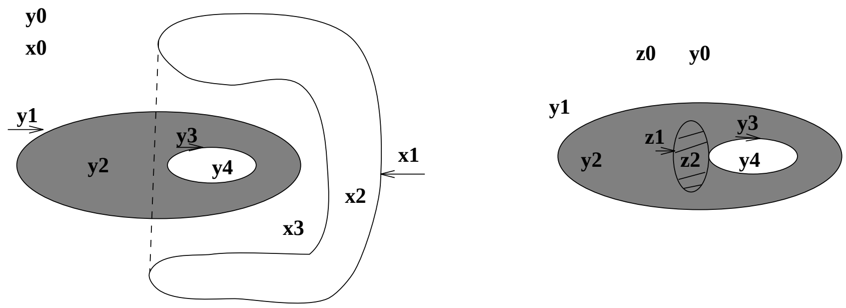


Figure 6: (a) Spatial relationships between a concave region x and a region with a hole y. (b) Spatial relationship between y and simple convex region z.

	y_0	y_1	y_2	y_3	y_4
x_0	1	1	1	0	0
x_1	1	0	0	0	0
x_2	1	0	0	0	0
x_3	1	1	1	1	1

(a)

	z_0	z_1	z_2
y_0	1	0	0
y_1	1	0	0
y_2	1	1	1
y_3	1	1	0
y_4	1	0	0

(b)

Figure 7: Intersection matrices for representing the spatial relationships in figure 6 respectively.

propagated. Rules 1 and 2 are the two general rules for propagating empty and non-empty intersections of components of spaces.

Note that in both rules the intermediate object (y) and its space components plays the main role in the propagation of intersections. Indeed, it shall be shown in the next example how the above two rules are applied a number of times equal to the number of components of the space of the intermediate object. Hence, the composition of spatial relations using this method becomes a tractable problem which can be performed in a definite limited number of steps.

2.3 Example of Spatial Reasoning with Complex Objects

The example in figure 6 is used for demonstrating the composition of relations using non-simple spatial objects. Figure 6(a) shows the relationship between a concave region x and a region with a hole y and 6(b) shows the relationship between object y and a simple convex region z where z touches the the hole in y. The intersection matrices corresponding to the two relationships are shown in figure 7(a) and 7(b) respectively.

Given that the possible set of relationships that can occur between x and z in a certain domain are as shown in figure 5, it is required to derive the possible relationships between these two objects given the situation in figure 6.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- y_0 intersections:

$$\{x_0, x_1, x_2, x_3\} \sqsupseteq y_0 \ \wedge \ y_0 \sqsubseteq \{z_0\} \quad \rightarrow \quad x_0 \cap z_0 \neq \phi \wedge x_1 \cap z_0 \neq \phi \wedge x_2 \cap z_0 \neq \phi \wedge x_3 \cap z_0 \neq \phi$$

- y_1 intersections:

$$\{x_0, x_3\} \sqsupseteq y_1 \ \wedge \ y_1 \sqsubseteq \{z_0\} \quad \rightarrow \quad x_1 \cap z_0 \neq \phi \wedge x_3 \cap z_0 \neq \phi$$

- y_2 intersections:

$$\{x_0, x_3\} \sqsupseteq y_2 \ \wedge \ y_2 \sqsubseteq \{z_0, z_1, z_2\} \quad \rightarrow \quad x_0 \cap (z_0 \cup z_1 \cup z_2) \neq \phi \wedge x_3 \cap (z_0 \cup z_1 \cup z_2) \neq \phi$$

- y_3 intersections:

$$\{x_3\} \sqsupseteq y_3 \ \wedge \ y_3 \sqsubseteq \{z_0, z_1\} \quad \rightarrow \quad x_3 \cap z_0 \neq \phi \wedge x_3 \cap z_1 \neq \phi$$

- y_4 intersections:

$$\{x_3\} \sqsupseteq y_4 \ \wedge \ y_4 \sqsubseteq \{z_0\} \quad \rightarrow \quad x_3 \cap z_0 \neq \phi$$

Applying rule 2 we get the following,

- $x_0 \sqsupseteq \{y_0, y_1, y_2\} \ \wedge \ \{y_0, y_1, y_2\} \sqsubseteq \{z_0, z_1, z_2\}$ x_0 has no empty intersections with components in Z.
- $x_1 \sqsupseteq y_0 \ \wedge \ y_0 \sqsubseteq \{z_0\} \rightarrow x_1 \cap z_1 = \phi \wedge x_1 \cap z_2 = \phi$
- $x_2 \sqsupseteq y_0 \ \wedge \ y_0 \sqsubseteq \{z_0\} \rightarrow x_2 \cap z_1 = \phi \wedge x_2 \cap z_2 = \phi$
- $x_3 \sqsupseteq \{y_0, y_1, y_2, y_3, y_4\} \ \wedge \ \{y_0, y_1, y_2, y_3, y_4\} \sqsubseteq \{z_0, z_1, z_2\}$ x_3 has no empty intersections with components in Z.

Refining the above constraints, we get the following intersection matrix.

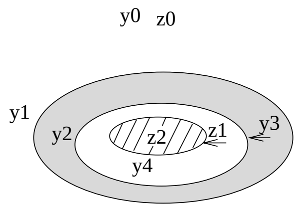
	z_0	z_1	z_2
x_0	1	-	a
x_1	1	0	0
x_2	1	0	0
x_3	1	1	a

Comparing the resulting matrix above with the matrices in figure 5, it can be seen that the result matrix corresponds to two possible relationships between objects x and z, namely the relationships R_3 and R_5.

A different conclusion is obtained if the relationship between objects y and z is as shown in figure 8(a) with a corresponding intersection matrix in 8(b). The composition of the relationships between x, y and z in this case will result in the definite matrix in figure 8(c) which corresponds to R_5 in figure 5.

3 Analysis of Related Issues

In this section the reasoning approach developed is going to be used to explain two interesting aspects of the spatial reasoning process, namely, when are the composition definite or indefinite and the process of conceptual neighbourhood. Also, the reasoning approach shall be shown to be applicable to reasoning in time.



(a)

	z_0	z_1	z_2
y_0	1	0	0
y_1	1	0	0
y_2	1	0	0
y_3	1	0	0
y_4	1	1	1

(b)

	z_0	z_1	z_2
x_0	1	0	0
x_1	1	0	0
x_2	1	0	0
x_3	1	1	1

(c)

Figure 8: Given the relationship between objects x and y as in figure 6 and 7(a) and the relation between the objects y and z as defined in (a) and (b) in this figure the composition shall result in the definite intersection matrix between x and z shown in (c).

3.1 Definite and Indefinite Compositions

As seen from the previous example indefinite compositions are those where the the result of the spatial reasoning problem is a set of disjunctive spatial relations as opposed to one definite relation. On the other hand definite compositions result in only one relation. In fact in composition tables, which hold the results of reasoning between all the possible set of relations between the concerned objects, many of the entries are disjunctive sets of spatial relations [13, 18]. Using Rule 1 $(x' \sqsupseteq y_j) \wedge (y_j \sqsubseteq z') \rightarrow (x' \cap z' \neq \phi)$, the following observations can be made.

1. If either x' or z' consist of only one element, then the rule shall propagate a definite set of intersections. For example, if y_j intersects the only element of x', then this element of x' must have a non-empty intersection with every element from the set z'. Also, if y_j intersects with the only element of z', then this element of z' must have a non-empty intersection with every element from the set x'. If this property holds for every component of the intermediate space Y then the composition must result in a definite relation. An example of this case is the composition of the inside relationship between two simple convex polygons : $inside(A, B) \wedge inside(B, C) \rightarrow inside(A, C)$

2. On the other hand, if at least one y_j of the space Y has a non-empty intersection with non-singleton sets x' and z', no definite intersections are propagated (i.e. $x' \cap z' \neq \phi$). If after the application of the reasoning rules this result still holds, then the composition shall produce a non-definite set of disjunctive relations.

3. If in rule 1 x' is X and z' is Z, i.e. $(X \sqsupseteq y_j) \wedge (y_j \sqsubseteq Z)$, no distinguishing constraints can be propagated from the component y_j, as this case is an expression of the first general constraint in section 2.2.1.

4. If in rule 1 x' is an infinite component and z' is an infinite component, then the rule becomes an expression of the second general constraint in section 2.2.1, i.e. no distinguishing constraint will be propagated.

5. If all the propagated intersections for the set of components of the intermediate space are either of type 3 or 4 above then the composition results in the universal relation (disjunction of set of all possible relationships) - since the only constraints propagated are the general ones. An example is the compositions: $overlap(A, B) \wedge overlap(B, C)$ and $disjoint(A, B) \wedge disjoint(B, C)$ for two simple convex polygons.

3.2 Conceptual Neighbourhood

An observation made by Freksa [19] on examining the temporal composition table derived by Allen [4] is that the table entries which are a disjunctive set of relations are always sets of relations which are conceptual neighbors. "Two relations between pairs of events are conceptual neighbors if they can be directly transformed into one another by continuous deformation (i.e., shortening or lengthening) of the events [17]". The same observation was made for the composition tables derived in the spatial domain [17] and this property was utilized in making the reasoning process more efficient.

However, there was no explanation on why this phenomenon occurs [10]. In this section the reasoning formalism developed shall be used to give an explanation on phenomenon of the conceptual neighbourhood.

When two spatial relationships are conceptual neighbors, the transition between the two relationships involves changing the intersection between one (or more) component x_i and another z_j in one relationship to the intersection between the same component x_i and another z_k in the other relationship where z_j is connected to z_k.

The initial assumptions in our formalism states that all the components of the objects and the space are dense and connected. Hence, from the rule 1, if $y_j \sqsubseteq x'$, and y_j is connected then all the elements of x' must be connected. The same applies to $y_j \sqsubseteq z'$ and hence the elements of z' must also be connected. Rule 1 states that $x' \cap z' \neq \phi$. This fact means that any element of x' can intersect only with connected element(s) of z' and vice versa.

In the case where the result of the composition is the possible intersection of one element x_i of x' to more than one element of z_j, z_k of z', the intersection possibilities (i.e., $(x_i \cap z_j \neq \phi) \vee (x_i \cap z_k \neq \phi) \vee (x_i \cap z_j \neq \phi \wedge x_i \cap z_k \neq \phi)$ must yield relationships which are conceptual neighbors since, z_j is connected to z_k.

The following example illustrates the above argument. Consider the composition of the relationships between simple convex regions in figure 9. By applying the reasoning rules we have that $x_1 \cap (z_1 \cup z_2) \neq \phi$ z_1 is connected to z_2 and thus the possible relationships from this composition are conceptual neighbors as shown in figure 10.

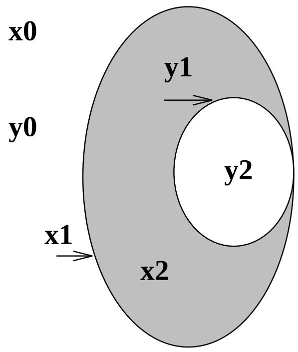
The only case where a composition may yield a disjunction of relationships which are not conceptual neighbors is when one or more components of the intermediate object are not connected. This case was given in Bennett [7] and is shown in figure 11 where it was used to illustrate an unexplained property of the conceptual neighbourhood.

3.3 Applying the Reasoning formalism in the Temporal Domain

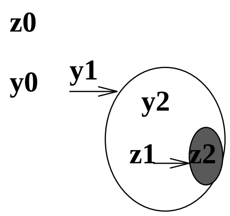
Consider an event e in an event space E as shown in figure 12. e can be decomposed into the following components: s: its start, f: its finish, t: its duration. The event space E is composed of e and p_0: a semi-infinite line representing the past of e and f_0: a semi infinite line representing the future of e. The connectivity matrix for E is as shown in figure 12(b).

The relationship between two events can be represented by an intersection matrix. For example the overlap relationship in figure 13 can be represented by the matrix in the same figure. Both the general space constraints in section 2.2.1 are also applicable in the temporal domain. In the above example, $f_{01} \cap f_{02} \neq \phi$ and $p_{01} \cap p_{02} \neq \phi$, i.e. the future as well as the past of any two events must intersect.

The two reasoning rules proposed are also applicable in the temporal domain. For example, consider the composition of the two relationships: $overlap(e1, e2)$ and $overlap(e2, e3)$ as shown in figure 13. Applying the two reasoning rules over the above matrices as in section 2.3, we get the result matrix in figure 14(a) which can corresponds to one of the three relations in figure 14(b). The 5 conclusions in 3.1 are also applicable in the temporal domain (e.g. the composition of $before(a, b)$ and $after(b, c)$ giving the universal relations as explained by conclusion 5).

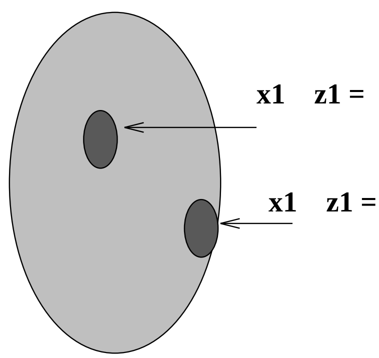


	y_0	y_1	y_2
x_0	1	0	0
x_1	1	1	0
x_2	1	1	1



	z_0	z_1	z_2
y_0	1	0	0
y_1	1	1	0
y_2	1	1	1

Figure 9: Covers relationship between x and y and between y and z.



	z_0	z_1	z_2
x_0	1	0	0
x_1	1	-	0
x_2	1	1	1

Figure 10: The result of the composition in figure 9.

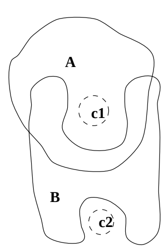
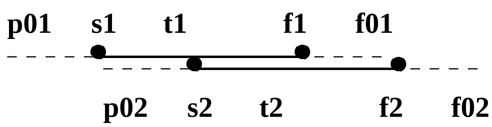


Figure 11: Discontinuous composition due to disconnected components [Bennet-94].

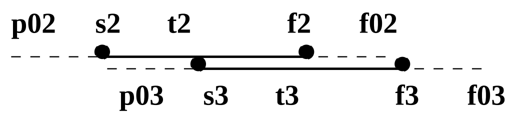
f_0				
1	f			
0	1	t		
0	0	1	s	
0	0	0	1	p_0

Figure 12: a) An event, b) representation of the event by adjacency matrix.



	f_{01}	f_1	t_1	s_1	p_{01}
f_{02}	1	0	0	0	0
f_2	1	0	0	0	0
t_2	1	1	1	0	0
s_2	0	0	1	0	0
p_{02}	0	0	1	1	1

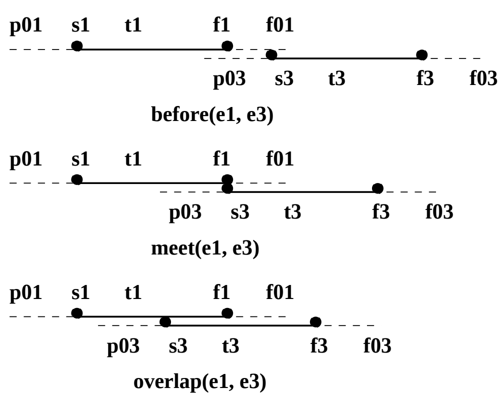
(a)



	f_{02}	f_2	t_2	s_2	p_{02}
f_{03}	1	0	0	0	0
f_3	1	0	0	0	0
t_3	1	1	1	0	0
s_3	0	0	1	0	0
p_{03}	0	0	1	1	1

(b)

Figure 13: (a) An overlap relationship between two events. (b) adjacency matrix corresponding to the relationship in (a).



	f_{01}	f_1	t_1	s_1	p_{01}
f_{03}	1	0	0	0	0
f_3	1	0	0	0	0
t_3	1	-	-	0	0
s_3	-	-	-	0	0
p_{03}	-	-	1	1	1

Figure 14: Result of the composition in figure 13 is a set of disjunctive relations $before(e_1, e_3)$, $meet(e_1, e_3)$ or $overlap(e_1, e_3)$.

4 Approaches to Spatial Reasoning

Two general approaches for deriving the composition of spatial relations can be identified, namely, *transitive propagation* and *theorem proving*.

- Transitive propagation: In this approach the transitive property of some spatial relations is utilized to carry out the required reasoning. This applies to the *order relations*, such as `before`, `after` and $(<, =, >)$ (for example, $a < b \ \wedge \ b < c \rightarrow a < c$), and to the subset relations such as `contain` and `inside` (for example, $inside(A, B) \wedge inside(B, C) \rightarrow inside(A, C)$, $east(A, B) \wedge east(B, C) \rightarrow east(A, C)$).

  Transitive property of the subset relations was employed by Egenhofer [13, 12] for reasoning over topological relationships. Transitive property of the order relations has been utilized by Mukerjee & Joe [24], Guesgen [20], Chang & Lu [9], Lee & Hsu [23] and Papadias & Sellis [26]. The latter proposed a picture algebra based on ordered relations of projection of objects.

  Although order relations can be utilized in reasoning over point-shaped objects, they cannot be directly applied when the actual shapes and proximity of objects are considered. In this case spatial factors such as shape, size, and proximity of the objects disrupt the strict order on which the precise reasoning is based and hence the derivation of the composition of spatial relationships is no longer a systematic process in this case.

- Theorem proving (elimination): where reasoning can be carried out by checking every relation in the full set of *sound* relations in the domain to see whether it is a valid consequence of the composition considered (theorems to be proved) and eliminating the ones which are not consistent with the composition.

  Bennett [8] have proposed a propositional calculus for the derivation of the composition of topological relations between simple regions using this method. However, checking each relation in the composition table to prove or eliminate is not possible in general cases and is considered a challenge for theorem provers [28].

In this paper the transitivity property of the subset relations is used for the development of the general spatial reasoning rules.

5 Conclusions

A general approach for spatial reasoning is proposed. The approach consists of a set of two general constraints to govern the spatial relationships between objects in space, and two general rules to propagate relationships between objects in space. The following conclusions may be drawn:

- The reasoning process is general and can be applied on any types of objects with random complexity.
- The approach is simple and is based on the application of two rules for the propagation of empty and non-empty intersections between object components.
- Conditions where definite and indefinite compositions result are identified.
- The reasoning method was used to explain the phenomenon of the conceptual neighbourhood.
- The approach was shown to be applicable in the representation and reasoning over events in the temporal domain.

Finally, the method is applied in a finite known number of steps (equal to the number of components of the intermediate objects) which allows its implementation in spatial information systems.

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