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SPARQS: a qualitative spatial reasoning engine

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Abstract

In this paper the design and implementation of a general qualitative spatial reasoning engine (SPARQS) is presented. Qualitative treatment of information in large spatial databases is used to complement the quantitative approaches to managing those systems, in particular, it is used for the automatic derivation of implicit spatial relationships and in maintaining the integrity of the database. To be of practical use, composition tables of spatial relationships between different types of objects need to be developed and integrated in those systems. The automatic derivation of such tables is considered to be a major challenge to current reasoning approaches. In this paper, this issue is addressed and a new approach to the automatic derivation of composition tables is presented. The method is founded on a sound set-theoretical approach for the representation and reasoning over arbitrarily shaped objects in space. A reasoning engine tool, SPARQS, has been implemented to demonstrate the validity of the approach. The engine is composed of a basic graphical interface where composition tables between the most common types of spatial objects are built. An advanced interface is also provided, where users are able to describe shapes of arbitrary complexity and to derive the composition of chosen spatial relationships. Examples of the application of the method using different objects and different types of spatial relationships are presented and new composition tables are built using the reasoning engine.

Keywords: Qualitative spatial reasoning; Qualitative spatial representation; Spatial composition tables

1. Introduction

Qualitative Spatial Representation and Reasoning (QSRR) is an active field of AI research where formalisms for encoding and manipulating qualitative spatial knowledge are studied [10]. A main aim of these techniques is the provision of tools to enhance the derivation and retrieval of implicit knowledge in large spatial databases typically used in applications such as, Geographic Information Systems (GIS), medical and biological databases and Computer Aided Design, Manufacture and Process Planning (CAD/CAM/CAPP). Such application domains are characterised by handling very large sets of spatial entities, relationships and constraints and their manipulation usually involve substantial computational costs. The ability to handle a certain level of indeterminacy makes QSRR techniques attractive in those domains. The goal is for such techniques to complement and enhance the traditional, usually computationally expensive, geometrical methods, especially when precise information are neither available nor needed. A simple example in a GIS is the derivation of the fact that the location of Peterhouse College is in the UK, from the facts that it is located in Cambridge and Cambridge is in the UK, without needing to execute a polygon-in-polygon geometric computation. Applications of QSRR include, qualitative spatial scene specification and scene feasibility problems, checking the similarity and consistency of data sets, integrating different spatial sets, and in initial pruning of search spaces in spatial query processing. Research is also ongoing for incorporating QSRR in the definition and implementation of spatial query languages. However, the qualitative approach has obvious limitations where useful characteristics of spatial objects such as shape and size are not used. Also, its application becomes limited when exact positions and tolerance constraints are considered. Hence, it can be argued that both the quantitative and qualitative approaches have complementary areas of strength and that any system which can combine the two paradigms in a way which uses their strength would be an effective platform for a range of novel and conventional applications. One approach to utilising QSRR in such systems is by the automatic development of composition or transitivity tables between different types of spatial objects. Reasoning can then be transformed into a simple process of table look-up to be invoked when needed. Several works have addressed this problem previously, and some

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composition tables between simple objects have been reported. These approaches are generally limited and applicable only in simple constrained domains. The problem is, however, recognised as a major challenge to automatic theorem provers [3,25], and no general solution has yet been found.

In this paper, a new approach is proposed for the automatic reasoning of spatial relationships and the automatic building of composition tables. The approach is novel as composition tables between objects of any type and complexity can be derived. The proposed method is implemented using java in the SPARQS (SPAtial Reasoning in Qualitative Space) reasoning engine tool. The engine is used to validate the method and demonstrate its generality. The paper is structured as follows. Section 2 outlines the representation and reasoning approach in topological spaces. The reasoning method is applied over topological relations between different types of objects. In Section 3 the reasoning engine is described and the process of building composition tables is illustrated. Section 4 provides an overview of related approaches and some conclusions are drawn in Section 5.

2. The formalism

This section addresses the problem of qualitative representation of objects with arbitrary spatial complexity and their topological relationships. The reasoning formalism is then presented, consisting of (a) general constraints to govern the spatial relationships between objects in space, and (b) general rules to propagate relationships between those objects. Both the constraints and the rules are based on a uniform representation of the topology of the objects, their embedding space and the representation of the relationships between them.

2.1. The general representation approach

Objects of interest and their embedding space are divided into components according to a required resolution. The connectivity of those components is explicitly represented. Spatial relations are represented by the intersection of object components [1] in a similar fashion to that described in Ref. [17] but with no restriction on object components to consist only of three parts (boundary, interior and exterior).

The topology of the object and the embedding space can then be described by a matrix whose elements represent the connectivity relations between its components. This matrix shall be denoted *adjacency matrix*. In the decomposition strategy, the complement of the object in question shall be considered to be infinite, and the suffix 0, e.g. (x_0) is used to represent this component.

Hence, the topology of a space S containing an object x is defined using the following equation

$$x = \bigcup_{i=1}^{n} x_i \tag{1}$$

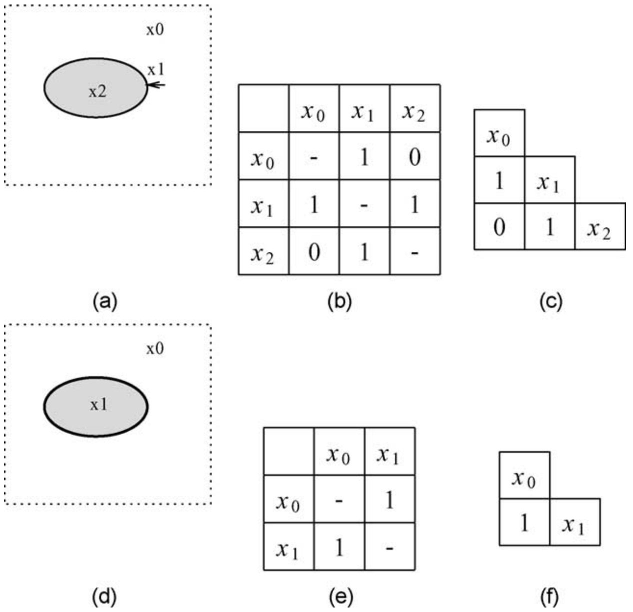
$$S_x = x \cup x_0 \tag{2}$$

where S_x is used to denote the space associated with object x.

In Fig. 1 different possible decompositions of a simple convex polygon and its embedding space is shown along with their adjacency matrices. In (a), the object is represented by two components, a linear component x_1 and an areal component x_2 and the rest of the space is represented by an infinite areal component x_0 representing the surrounding area. In (d), only one areal component is used to represent the polygon. Both representations are valid and may be used in different contexts. Different decomposition strategies for the objects and their embedding spaces can be used according to the precision of the relations required and the specific application considered. The higher the resolution used (or the finer the components of the space and the objects), the higher the precision of the resulting set of relations in the domain considered.

The fact that two components are connected is represented by a (1) in the adjacency matrix and by a (0) otherwise. Since connectivity is a symmetric relation, the resulting matrix will be symmetric around the diagonal. Hence, only half the matrix is sufficient for the representation of the object's topology and the matrix can be collapsed to the structure in Fig. 1(c) and (f).

Semi-bounded areas of the embedding space can also be represented (as virtual components) if needed. For example,



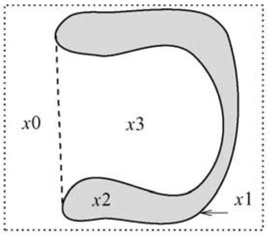
	x_0	x_1	x_2
x_0	-	1	0
x_1	1	-	1
x_2	0	1	-

x_0		
1	x_1	
0	1	x_2

	x_0	x_1
x_0	-	1
x_1	1	-

x_0	
1	x_1

Fig. 1. (a,d) Possible decompositions of a simple convex polygon and its embedding space. (b,e) Adjacency matrices corresponding to the two shapes in (a) and (d), respectively. (c,f) Half the symmetric adjacency matrix is sufficient to capture the object representation.



(a)

x_0			
1	x_1		
0	1	x_2	
1	1	0	x_3

(b)

Fig. 2. (a) Using virtual components to represent semi-bounded components in space. (b) Adjacency matrix for the shape in (a).

Fig. 2(a) shows a possible decomposition of a concave shaped object and its embedding space. In (b) the adjacency matrix for its components is presented. The object is represented by two components a linear component x_1 and an areal component x_2 and the rest of its embedding space is represented by a finite areal component x_3 (representing the virtual enclosure) and infinite areal component x_0 representing the surrounding area.

2.1.1. The underlying representation of spatial relations

In this section, the representation of the topological relations through the intersection of their components is adopted and generalized for objects of arbitrary complexity.

Distinction of topological relations is dependent on the strategy used in the decomposition of the objects and their related spaces. For example, in Fig. 3 different relationships between two objects x and y are shown, where in (a) x is outside y and in (b) x is inside y. Object y is decomposed into two components y_1 and y_2 and the rest of the space associated with y is decomposed into two components: y_3 representing the enclosure and y_0 representing the rest of the space. Note that it is the identification of the (virtual) component y_3 that makes the distinction between the two relationships in the figure. The complete set of spatial relationships are identified by combinatorial intersection of the components of one space with those of the other space.

If $R(x,y)$ is a relation of interest between objects x and y, and X and Y are the spaces associated with the objects, respectively, such that m is the number of components in X and l is the number of components in Y, then a spatial relation $R(x,y)$ can be represented by one instance of the following equation:

$$R(x,y) = X \cap Y = \left(\bigcup_{i=1}^{m} x_i\right) \cap \left(\bigcup_{j=1}^{l} y_j\right)$$

$$= (x_1 \cap y_1, \ldots, x_1 \cap y_l, x_2 \cap y_1, \ldots, x_m \cap y_l)$$

The intersection $x_i \cap y_j$ can be an empty or a non-empty intersection. The above set of intersections shall be represented by an intersection matrix, as follows

$$R(x,y) = \begin{array}{c|c|c|c|c} & y_0 & y_1 & y_2 & \cdots \\ \hline x_0 & & & & \\ \hline x_1 & & & & \\ \hline x_2 & & & & \\ \hline \vdots & & & & \end{array}$$

For example, the intersection matrices corresponding to the spatial relationships in Fig. 3 are shown in Fig. 4. The components x_1 and x_2 have a non-empty intersection with y_0 in Fig. 4(a) and with y_3 in Fig. 4(b). Different combinations in the intersection matrix can represent different qualitative relations. The set of valid or sound spatial relationships between objects is dependent on the particular domain studied.

2.2. The general reasoning formalism

The reasoning approach consists of: (a) general constraints to govern the spatial relationships between objects in space, and (b) general rules to propagate relationships between the objects.

2.2.1. General constraints

The intersection matrix is in fact a set of constraints whose values identifies specific spatial relationships. The process of spatial reasoning can be defined as the process of propagating the constraints of two spatial relations (for example, $R_1(A,B)$ and $R_2(B,C)$), to derive a new set of constraints between

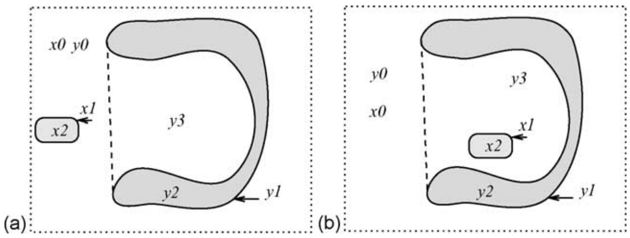


Fig. 3. Different qualitative spatial relationships can be distinguished by identifying the appropriate components of the objects and the space.

(a)

	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	1	0	0	0
x_2	1	0	0	0

(b)

	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	0	0	0	1
x_2	0	0	0	1

Fig. 4. The corresponding intersection matrices for the relationships in Fig. 3, respectively.

objects. The derived constraints can then be mapped to a specific spatial relation (i.e. the relation $R_3(A, C)$).

A subset of the set of constraints defining all possible spatial relations are general and applicable to any relationship between any objects. These general constraints are a consequence of the initial assumptions used in the definition of the object and space topology. The two general constraints are:

1. Every unbounded (infinite) component of one space must intersect with at least one unbounded (infinite) component of the other space.

   Intuitively this rule says that it is impossible for an infinite component in the space to only have an intersection with finite component(s). In this case the infinite component becomes a subset of the finite component(s) which is not possible.
2. Every component from one space must intersect with at least one component from the other space.

   If one component of one space does not intersect with any component of the other space, either the two spaces are not equal or the spaces are not dense or *connected*. Both conditions are excluded by the initial assumptions. This implies that there cannot exist a row or a column in the intersection matrix whose elements are all empty intersections, hence the combinatorial cases in the matrix where this case exists can be ignored.

2.2.2. General reasoning rules

Composition of spatial relations is the process through which the possible relationship(s) between two object x and z is derived given two relationships: R_1 between x and y and R_2 between y and z. Two general reasoning rules for the propagation of intersection constraints are presented. The rules are characterized by the ability to reason over spatial relationships between objects of arbitrary complexity in any space dimension. These rules allow for the automatic derivation of the composition (transitivity) tables between any spatial shapes.

2.2.2.1. Reasoning rules. Composition of spatial relations using the *intersection* representation approach is based on the transitive property of the subset relations. In what follows the following subset notation is used. If x' is a set of components (set of point-sets) $\{x_1, \ldots, x_{m'}\}$ in a space X, and y_j is a component in space Y, then $\sqsubseteq$ denotes the following subset relationship.

- $y_j \sqsubseteq x'$ denotes the subset relationship such that: $\forall x_i \in x' (y_j \cap x_i \neq \phi) \wedge y_j \cap (X - x_1 - x_2 \cdots - x_m) = \phi$ where $i = 1, \ldots, m'$. Intuitively, this symbol indicates that the component y_j intersects with every set in the collection x' and does not intersect with any set outside of x'.

If x_i, y_j and z_k are components of objects x, y and z respectively, then if there is a non-empty intersection between x_i and y_j, and y_j is a subset of z_k, then it can be concluded that there is also a non-empty intersection between x_i and z_k.

$$(x_i \cap y_j \neq \phi) \wedge (y_j \subseteq z_k) \rightarrow (x_i \cap z_k \neq \phi)$$

This relation can be generalized in the following two rules. The rules describe the propagation of intersections between the components of objects and their related spaces involved in the spatial composition.

Rule 1: propagation of non-empty intersections. Let $x' = \{x_1, x_2, \ldots, x_{m'}\}$ be a subset of the set of components of space X whose total number of components is m and $m' \leq m$; $x' \subseteq X$. Let $z' = \{z_1, z_2, \ldots, z_{n'}\}$ be a subset of the set of components of space Z whose total number of components is n and $n' \leq n$; $z' \subseteq Z$. If y_j is a component of space Y, the following is a governing rule of interaction for the three spaces X, Y and Z.

$$\begin{aligned}(x' \sqsupseteq y_j) \wedge (y_j \sqsubseteq z') \rightarrow (x' \cap z' \neq \phi) \equiv (x_1 \cap z_1 \\ \neq \phi \vee \cdots \vee x_1 \cap z_{n'} \neq \phi) \wedge (x_2 \cap z_1 \neq \phi \vee \cdots \vee x_2 \cap z_{n'} \\ \neq \phi) \wedge \cdots \wedge (x_{m'} \cap z_1 \neq \phi \vee \cdots \vee x_{m'} \cap z_{n'} \neq \phi)\end{aligned}$$

The above rule states that if the component y_j in space Y has a positive intersection with every component from the sets x' and z', then each component of the set x' must intersect with at least one component of the set z' and vice versa.

The constraint $x_i \cap z_1 \neq \phi \vee x_i \cap z_2 \neq \phi \cdots \vee x_i \cap z_{n'} \neq \phi$ can be expressed in the intersection matrix by a label, for example, the label a_r ($r = 1$ or 2) in the following matrix indicates $x_1 \cap (z_2 \cup z_4) \neq \phi$ (x_1 has a positive intersection with z_2, or with z_4 or with both). A- in the matrix indicates that the intersection is either positive or negative

	z_1	z_2	z_3	z_4	$\cdots$	z_n
x_1	$-$	a_1	$-$	a_2	$-$	$-$

Rule 1 represents the propagation of non-empty intersections of components in space. A different version of the rule for the propagation of empty intersections can be stated as follows.

Rule 2: propagation of empty intersections. Let $z' = \{z_1, z_2, \ldots, z_{n'}\}$ be a subset of the set of components of space Z whose total number of components is n and $n' < n$; $z' \subset Z$. Let $y' = \{y_1, y_2, \ldots, y_{l'}\}$ be a subset of the set of components of space Y whose total number of components is l and $l' < l$; $y' \subset Y$. Let x_i be a component of the space X. Then the following is a governing rule for the spaces X, Y and Z.

$$(x_i \sqsubseteq y') \wedge (y' \sqsubseteq z') \rightarrow (x_i \cap (Z - z_1 - z_2 \cdots - z_{n'}) = \phi)$$

Rules 1 and 2 are the two general rules for propagating empty and non-empty intersections of components of spaces. Note that in both rules the intermediate object (y) and its space components plays the main role in the propagation of intersections. The first rule is applied a number of times equal to the number of components of the space of the intermediate object. Hence, the composition of spatial relations becomes a tractable problem which can be performed in a defined limited number of steps.

2.2.2.2. Soundness and completeness of the formalism. The formalism can be said to be sound if any derived conclusion using the rules follows set-theoretically, and the formalism can be said to be complete if any conclusions which follows semantically from the axioms of the set theory are also derivable by the formalism.

In this section the formalism is proved to be sound and complete using the basic axioms of transitivity and set intersections in the set theory, in particular

- transitivity of subsets: $A \subseteq B \subseteq C \rightarrow A \subseteq C$, and its implication: $A \subseteq C \rightarrow A \cap (C^*) = \phi$, where C^* is the complement of C.
- set intersection: $A \cap B \wedge B \subseteq C \rightarrow A \cap C \neq \phi$, and, $C \cap B \wedge B \subseteq A \rightarrow A \cap C \neq \phi$. These rules can be derived directly from the transitivity axiom as follows: If $\exists \alpha(\alpha \in A \wedge \alpha \in B)$ then $(\alpha \subseteq A) \wedge (\alpha \subseteq B) \wedge (B \subseteq C) \rightarrow \alpha \subseteq C$ or $\alpha \cap C \neq \phi$. Hence, $A \cap B \wedge B \subseteq C \rightarrow A \cap C \neq \phi$.

Soundness of the formalism. Rule 1 states that

$$\begin{aligned}(x' \sqsupseteq y_j) \wedge (y_j \sqsubseteq z') \rightarrow (x' \cap z' \neq \phi) \equiv (x_1 \cap z_1 \\ \neq \phi \vee \cdots \vee x_1 \cap z_{n'} \neq \phi) \wedge \cdots \wedge (x_{m'} \cap z_1 \\ \neq \phi \vee \cdots \vee x_{m'} \cap z_{n'} \neq \phi) z_1 \cap x_1 \neq \phi \vee \cdots \vee z_1 \cap x_{m'} \\ \neq \phi) \wedge \cdots \wedge (z_{n'} \cap x_1 \neq \phi \vee \cdots \vee z_{n'} \cap x_{m'} \neq \phi)\end{aligned}$$

i.e. $(x_1 \cap z' \neq \phi, x_{m'} \cap z' \neq \phi, z_1 \cap x' \neq \phi, z_{n'} \cap x' \neq \phi)$

Since $x' \sqsupseteq y_j \rightarrow (y_j \cap x_1 \neq \phi \wedge y_j \cap x_2 \neq \phi \wedge \cdots \wedge y_j \cap x_{m'} \neq \phi)$, and, $z' \sqsupseteq y_j \rightarrow (y_j \cap z_1 \neq \phi \wedge y_j \cap z_2 \neq \phi \wedge \cdots \wedge y_j \cap z_{n'} \neq \phi)$, then, Rule 1 can be expressed by the collection of the following axioms:

$$\begin{gathered}x_1 \cap y_j \neq \phi \wedge y_j \sqsubseteq z' \rightarrow x_1 \cap z' \neq \phi \\ x_2 \cap y_j \neq \phi \wedge y_j \sqsubseteq z' \rightarrow x_2 \cap z' \neq \phi \\ \vdots \\ x_{m'} \cap y_j \neq \phi \wedge y_j \sqsubseteq z' \rightarrow x_{m'} \cap z' \neq \phi \\ z_1 \cap y_j \neq \phi \wedge y_j \sqsubseteq x' \rightarrow z_1 \cap x' \neq \phi \\ z_2 \cap y_j \neq \phi \wedge y_j \sqsubseteq x' \rightarrow z_2 \cap x' \neq \phi \\ \vdots \\ z_{n'} \cap y_j \neq \phi \wedge y_j \sqsubseteq x' \rightarrow z_{n'} \cap x' \neq \phi\end{gathered}$$

Hence, Rule 1 reduces to the axiom of set intersection and is therefore sound.

Rule 2 states that:

$$(x_i \sqsubseteq y') \wedge (y' \sqsubseteq z') \rightarrow (x_i \cap (Z - z') = \phi)$$

$Z - z'$ is the complement of z'. Using the transitivity of subsets, $x_i \sqsubseteq y' \wedge y' \sqsubseteq z' \rightarrow x_i \subseteq z'$, then intersection of x_i with the complement of z' must be empty. Hence Rule 2 is also sound.

Completeness of the formalism. As shown above, Rule 1 is an equivalent form of the set intersection axiom and hence any conclusion which can be derived using this axiom is also derivable using Rule 1.

From the set theory we have that: $A \subseteq B \subseteq C \rightarrow A \subseteq C \rightarrow A \cap (U - C) = \phi$, where U is the universal set for space. In the formalism the underlying spaces for the objects are equal, i.e. $X = Y = Z$ and all are equivalent to the Universal set for space. Hence, $\forall x \in X(x \subseteq Z)$, and similarly, $\forall z \in Z(z \subseteq X)$. From Rule 2 we have that: $x_i \sqsubseteq y' \sqsubseteq z' \rightarrow x_i \subseteq z' \rightarrow x_i \cap (Z - z') = \phi$ where Z is the universal set for space. Then Rule 2 reduces to the subset transitivity axiom and its implication, and any conclusion which can be derived using these axioms are also derivable by the formalism.

Since both rules in the formalism are equivalent to basic axioms of the set theory, then the formalism is set-theoretically complete with respect to the two axioms and any axioms derived from them.

2.3. Analysis of the formalism

If m' and n' are the number of components of the sets x' and z', respectively, and m and n are the total number of components of the spaces X and Z, respectively, and $x' \sqsubseteq X$ and $z' \sqsubseteq Z$. Using Rule 1 the composition of relations can be classified into the following.

(I) If $(m' = 1 \vee n' = 1)$, (e.g. $x' = \{x_1\}$ or $z' = \{z_1\}$ or both) then the rule shall propagate a definite set of intersections. For example, if y_j intersects with the only element of x', then this element of x' must have a non-empty intersection with every element from the set z'. Also, if y_j intersects with the only element of z', then this element of z' must have a non-empty intersection with every element from the set x'. If this property holds for every component of the intermediate space y then the composition must result in a definite relation. An example of this case is the composition of the inside relationship between two simple convex regions.

(II) If $(m' > 1 \wedge n' > 1)$, (e.g. if $x' = \{x_1, x_2\}$ and $z' = \{z_1, z_2\}$), for at least one y_j of the space Y, no definite intersections are propagated (i.e. $x' \cap z' \neq \phi$). If after the application of the reasoning rules this result still holds, then the composition shall produce a set of disjunctive relations.

(III) If $(m' = m \wedge n' = n)$, i.e. $(X \sqsupseteq y_j) \wedge (y_j \sqsubseteq Z)$, no distinguishing constraints can be propagated from the component y_j, as this case is an expression of the second general constraint. Also, since the implication of such constraint is that every component of one space may intersect with all the components of the other space no empty intersection will be propagated (using Rule 2) for any component.

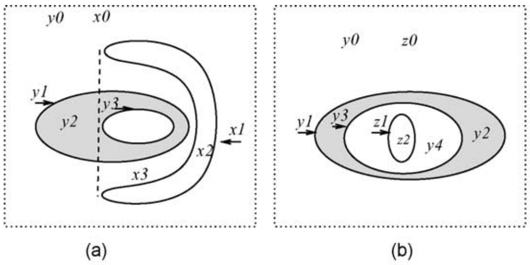
(IV) If $(m' = 1 \wedge n' = 1 \wedge x' = \{x_0\} \wedge z' = \{z_0\})$, i.e. x' is the infinite component and z' is the infinite component, then the rule becomes an expression of the first general constraint, i.e. no distinguishing constraint will be propagated.

(V) If all the propagated intersections for the set of components of the intermediate space are either of type III or IV above or both then the composition results in the universal relation (disjunction of all possible relationships)—since the only constraints propagated are the general ones, i.e. no specific constraint is propagated.

2.4. Example 1: propagation of definite relations

The example in Fig. 5 is used for demonstrating the composition of relations using non-simple spatial objects. Fig. (a) shows the relationship between a concave polygon x and a polygon with a hole y and (b) shows the relationship between object y and a simple polygon z where z touches the hole in y. The intersection matrices corresponding to the two relationships are also shown.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows.



	y_0	y_1	y_2	y_3	y_4
x_0	1	1	1	0	0
x_1	1	0	0	0	0
x_2	1	0	0	0	0
x_3	1	1	1	1	1

(c)

	z_0	z_1	z_2
y_0	1	0	0
y_1	1	0	0
y_2	1	0	0
y_3	1	0	0
y_4	1	1	1

(d)

Fig. 5. (a,b) Spatial relationships between non-simple objects x, y and z. (c,d) Corresponding intersection matrices.

From Rule 1 we have

- y_0 intersections:

$$\{x_0, x_1, x_2, x_3\} \sqsupseteq y_0 \wedge y_0 \sqsubseteq \{z_0\} \rightarrow x_0 \cap z_0 \neq \phi \wedge x_1 \cap z_0 \neq \phi \wedge x_2 \cap z_0 \neq \phi \wedge x_3 \cap z_0 \neq \phi$$

- y_1 intersections:

$$\{x_0, x_3\} \sqsupseteq y_1 \wedge y_1 \sqsubseteq \{z_0\} \rightarrow x_0 \cap z_0 \neq \phi \wedge x_3 \cap z_0 \neq \phi$$

- y_2 intersections:

$$\{x_0, x_3\} \sqsupseteq y_2 \wedge y_2 \sqsubseteq \{z_0\} \rightarrow x_0 \cap z_0 \neq \phi \wedge x_3 \cap z_0 \neq \phi$$

- y_3 intersections:

$$\{x_3\} \sqsupseteq y_3 \wedge y_3 \sqsubseteq \{z_0\} \rightarrow x_3 \cap z_0 \neq \phi$$

- y_4 intersections:

$$\{x_3\} \sqsupseteq y_4 \wedge y_4 \sqsubseteq \{z_0, z_1, z_2\} \rightarrow x_3 \cap z_0 \neq \phi \wedge x_3 \cap z_1 \neq \phi \wedge x_3 \cap z_2 \neq \phi$$

Refining the above constraints, we get the following intersection matrix

	z_0	z_1	z_2
x_0	1	0	0
x_1	1	0	0
x_2	1	0	0
x_3	1	1	1

The resulting matrix corresponds to one possible relationship between x and z as shown in Fig. 6.

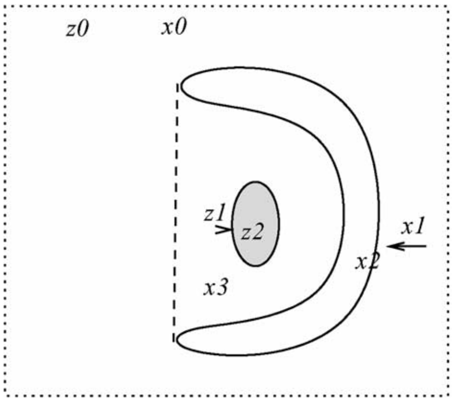
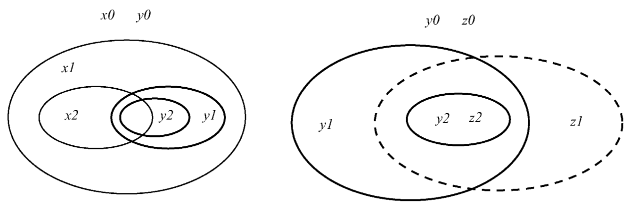


Fig. 6. Resulting definite relation from the composition of the relations in Fig. 5.



(a) (b)

	y_0	y_1	y_2
x_0	1	0	0
x_1	1	1	1
x_2	1	1	1

(c)

	z_0	z_1	z_2
y_0	1	1	0
y_1	1	1	0
y_2	0	0	1

(d)

Fig. 7. (a,b) Spatial relationships between vague regions x, y and z. (c,d) Corresponding intersection matrices.

2.5. Example 2: propagation of indefinite relations

The example in Fig. 7 is used for demonstrating the composition of relations using non-simple spatial objects, resulting in a set of possible relationships between objects x and z.The figure shows example relationships and the corresponding intersection matrices, between regions with indeterminate boundaries x and y and z as defined in Ref. [7].The problem of representing vague regions have been addressed in various research works previously [28].In Ref. [28] a set of 44 possible relations is defined between the two region with broad boundaries. The following is an example of how the reasoning rules are applied to derive the composition of two example relations.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From Rule 1 we have

- y_1 intersections:

$$\{x_1, x_2\} \sqsupseteq y_1 \wedge y_1 \sqsubseteq \{z_1, z_0\} \rightarrow x_1 \cap (z_1 \cup z_0) \neq \phi \wedge x_2 \cap (z_1 \cup z_0) \neq \phi$$

- y_2 intersections:

$$\{x_1, x_2\} \sqsupseteq y_2 \wedge y_2 \sqsubseteq \{z_2\} \rightarrow x_1 \cap z_2 \neq \phi \wedge x_2 \cap z_2 \neq \phi$$

- y_0 intersections:

$$\{x_1, x_2, x_0\} \sqsupseteq y_0 \wedge y_0 \sqsubseteq \{z_1, z_0\} \rightarrow x_1 \cap (z_1 \cup z_0) \neq \phi \wedge x_2 \cap (z_1 \cup z_0) \neq \phi \wedge x_0 \cap (z_1 \cup z_0) \neq \phi$$

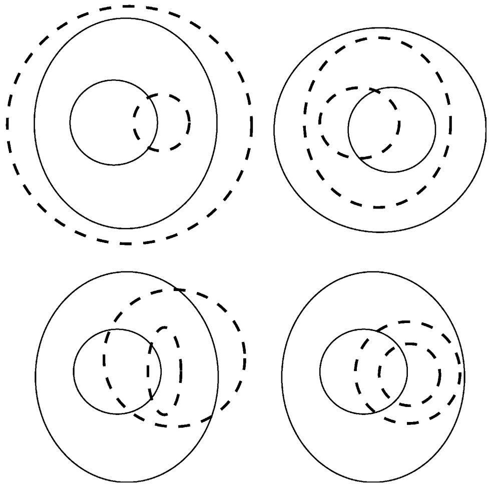


Fig. 8. Possible configurations for the composition in Fig. 7.

Refining the above constraints, we get the following intersection matrix

	z_1	z_2	z_0
x_1	a_1, c_1	1	a_2, d_1
x_2	b_1, c_2	1	b_2, d_2
x_0	?	0	1

where a_1 and a_2 represent the constraint $x_1 \cap (z_1 \vee z_0) = 1$ and b_1 and b_2 represent the constraint $x_2 \cap (z_1 \vee z_0) = 1$, c_1 and c_2 represent the constraint $z_1 \cap (x_1 \vee x_2) = 1$ and d_1 and d_2 represent the constraint $z_0 \bigcap (x_1 \vee x_2) = 1$ and the ? represents $(1 \vee 0)$. The result matrix corresponds to one of four possible relationships between x and z, namely numbers 21–23 and 25, as shown in Fig. 8.

3. SPARQS: the reasoning engine

To demonstrate the validity of the proposed approach, a reasoning engine has been designed and implemented using java. The interface to the program, named SPARQS consists of two parts. A basic interface is provided, where the topology of some common spatial object shapes are predefined, as shown in Fig. 9(a). Users are able to choose object types from a menu of available ones, namely, points, lines, simple regions, regions with indeterminate boundaries and concave regions. Users are then offered a selection of possible topological spatial relationships between the chosen object types. Sets of relationships are shown graphically and categorised using a coarse classification scheme under four headings, namely, disjoint, inside, overlap and touch to enhance the usability of the interface. The reasoning rules are applied to propagate the intersection matrices and produce the result matrix. The constraints in

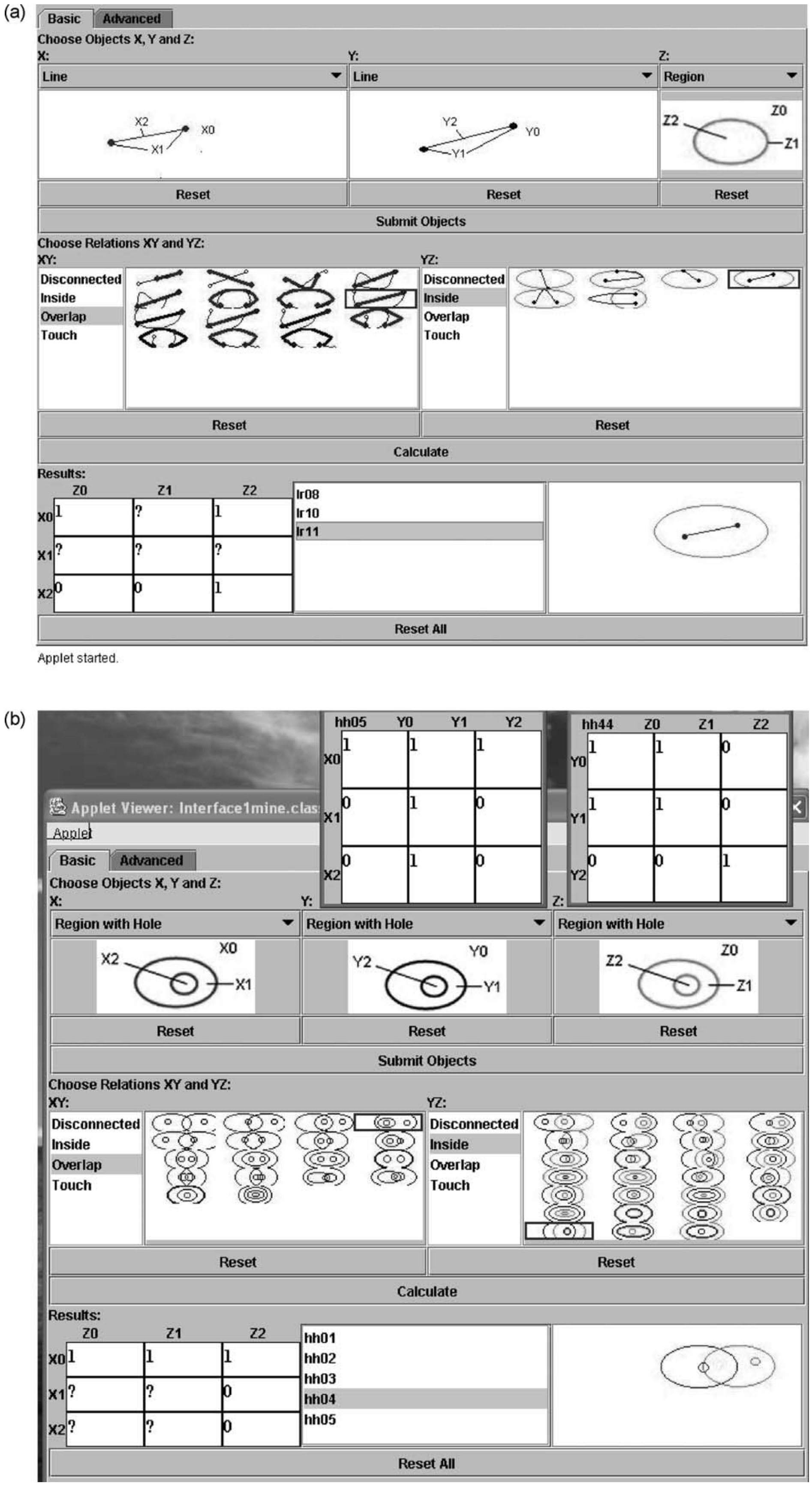


Fig. 9. The basic interface in SPARQS. (a) Composition of relationships between lines and region. (b) Composition of relationships between regions with holes.

the matrix are then matched to the set of possible relationships and all the ones satisfying the constraints are displayed in the result window, as shown in (a). The program is flexible where the input spatial relationships can be changed and resubmitted and the result re-calculated, as shown in (b).

A preliminary implementation of an advanced interface is also provided as shown in Fig. 10. Users are able to fill in adjacency and intersection matrices, which are subsequently used by the system to derive the resulting relationships. Some validation checks are done on the input matrices, e.g. to reject matrices that violate the general constraints described earlier, where no rows or columns in the matrix are allowed to contain only zeros. The result constraint matrix is therefore dependent on the validity of the input shapes and relations. Enhancement to the interface may be possible, where a more guided approach to input, possibly using sketch-based techniques, can be utilised to ensure valid entries.

The engine has been used to derive new composition tables between all the combination of objects defined in the basic interface, e.g. between simple regions, concave regions and regions with indeterminate boundaries, etc. Part of the composition table between regions with indeterminate boundaries is shown in Tables 2 and 3. The full set of 44 sound relations between those regions are as defined in Ref. [28] and are shown in Table 1.

Note the notation of the result of the compostions in Tables 2 and 3. An example is shown in Fig. 11, where the matrix of dots represent the 44 relations, read from left to right and from top to bottom. A black dot indicates that the existence of the relation. Hence, the example in the figure denotes that the result of the composition is R_1 or R_2 or R_6 or R_7 or R_8.

4. Comparison with related work

The main advantage of the method proposed above is its generality. The same methodology is used for the definition of simple, complex, composite regions, as well as regions with indeterminate boundaries. The method is also adaptable, where different levels of representation can be devised by hiding or revealing the details of objects as required. The method is therefore well adapted for use as a basis for a spatial reasoning formalism.

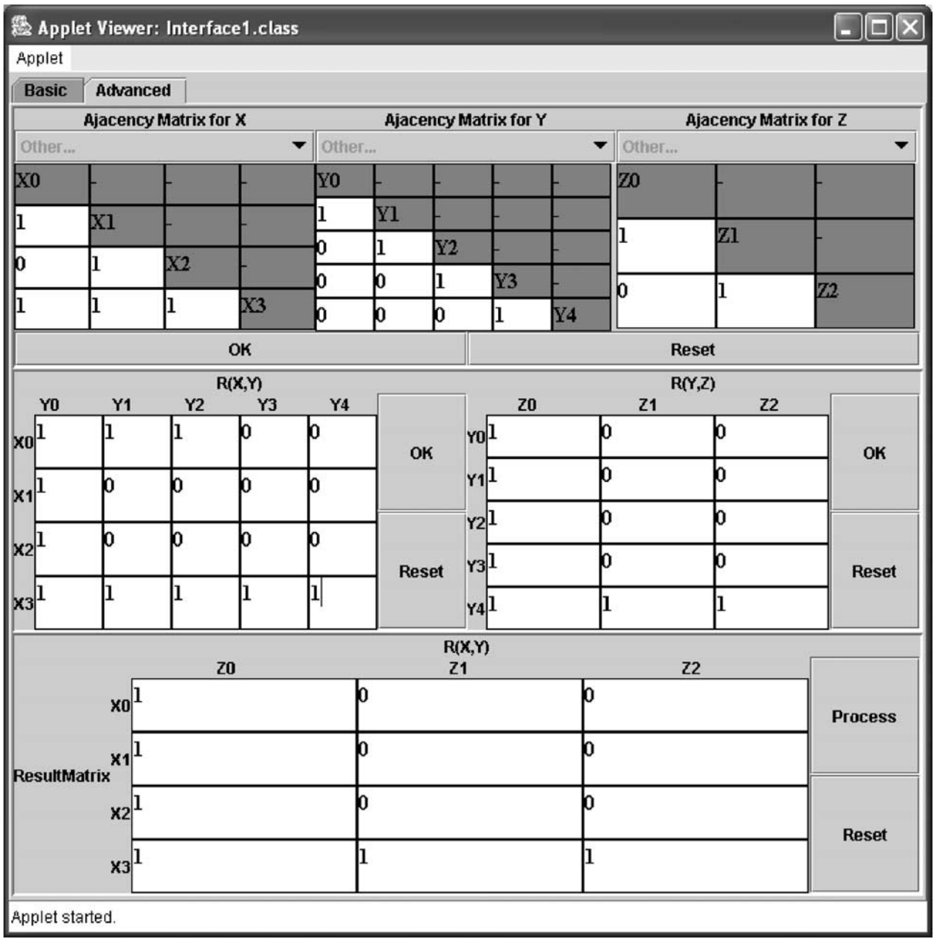
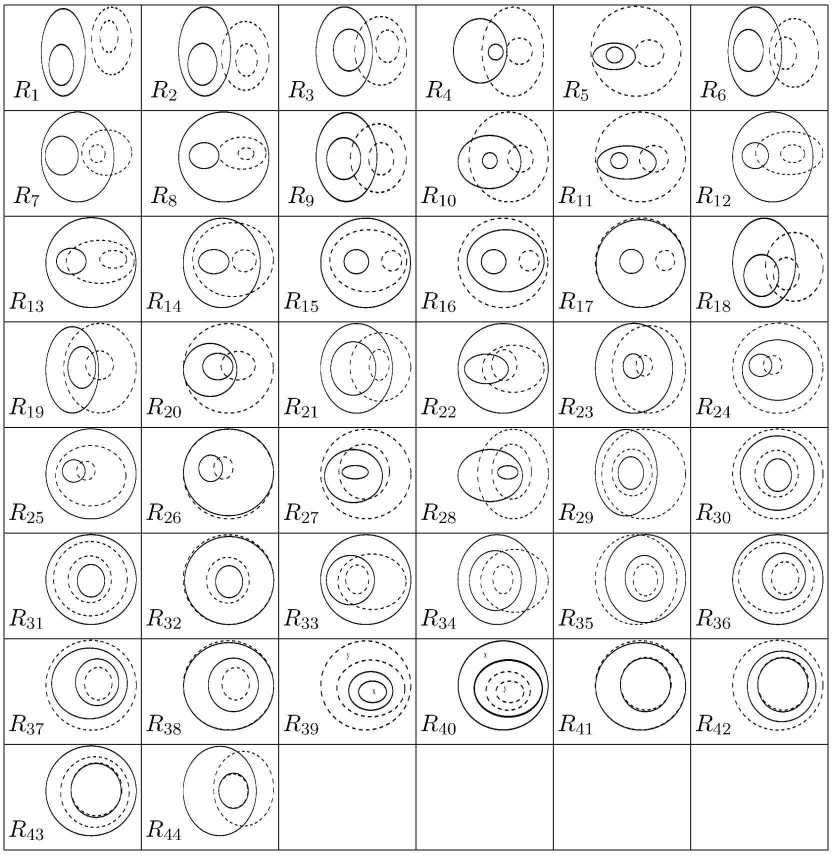


Fig. 10. The advanced interface in SPARQS. Users specify the adjacency and intersection matrices. The example in the figure corresponds to the Example 1.

Table 1
The set of 44 sound topological relations between regions with broad boundaries as defined in Ref. [7]



Representing complex regions has been addressed in many works. Cohn et al. [10,11] extended a logic-based formalism to handle concave regions, and regions with holes (doughnut shapes). New axioms and theories had to be devised to define the new shapes. The main drawback of this approach is its complexity, as new, possibly considerable, extensions of the formalism have to be devised with every new shape considered.

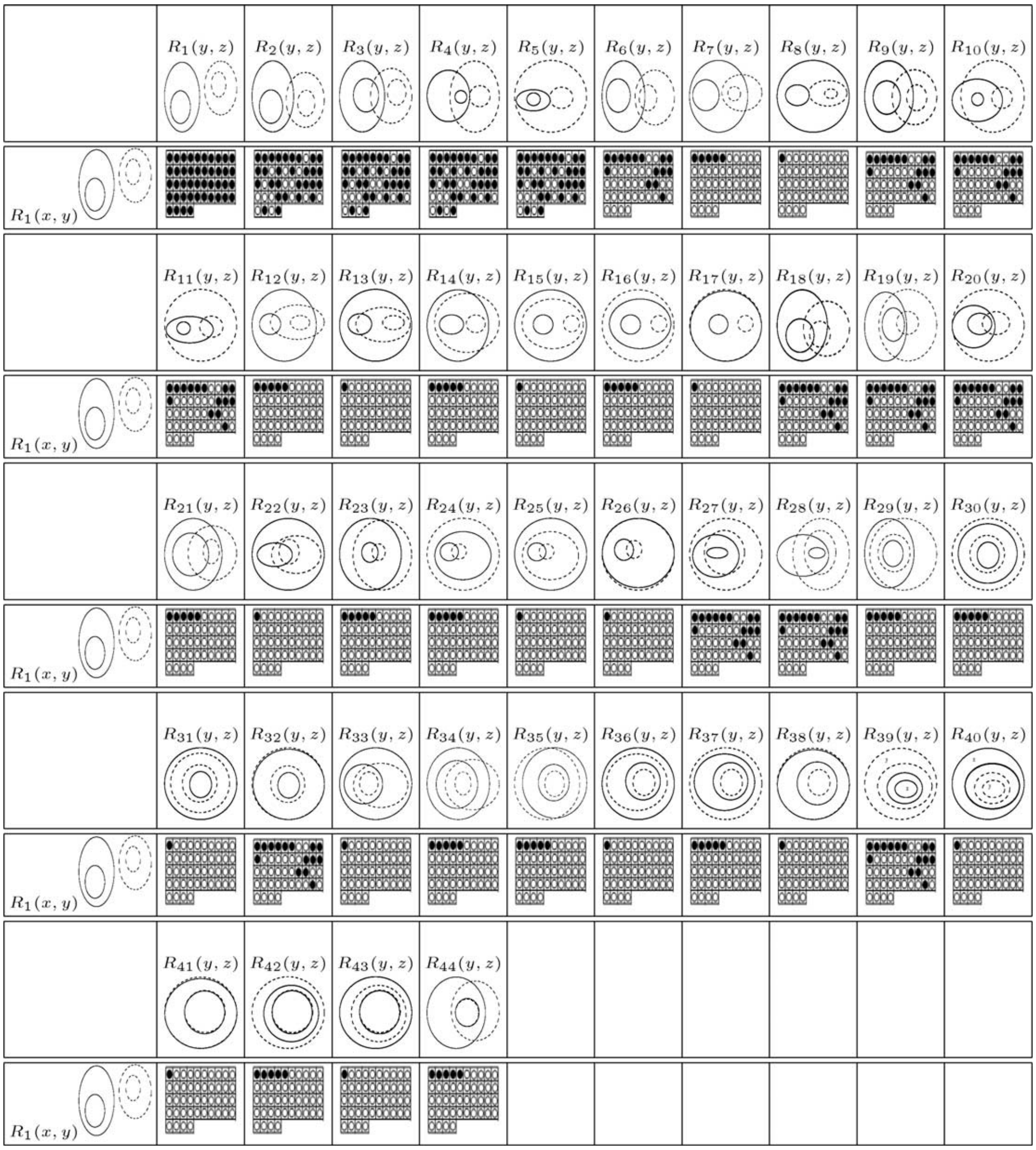
Egenhofer et al. [17,16] used point-set topology to define simple regions, using three components, boundary, interior and exterior. The method proposed here deviates from their work in one important respect, which has far-reaching implications. The constraint on the object components has been relaxed to be any possible set of components, which satisfies the main assumptions behind the formalism. The notions of boundaries, interiors and exteriors were dropped and the notion of object and space components is used instead. Other methods were devised in Ref. [15] to define regions with holes, through the definition of spatial relationships between simple regions and no extension for the method was proposed for the definition of irregular or concave regions.

The work of Clementini and De Felice [7] follows closely the method of Egenhofer, and provides a definition for regions with holes using boundaries, interiors and exteriors. Their method inherits the same limitations of Ref. [17]. In another work [8], Clementini et al. addressed the issue of defining composite regions for use in spatial query languages, by defining explicit relationships between all the components in the object, in the same way, regions with holes were defined in Ref. [15].

Clementini et al. [6] proposed a method of representing unique topological relationships between two composite regions (composed from simple regions without holes) as a set of rules which use only binary topological relationships at component level to decide the topological relationship between complex objects at higher level. The work of Nguyen et al. [23] follows a similar approach to the above, but generalises the rules for connected composite objects with or without holes.

Coenen and Pepijn [9] proposed an ontology for objects and relationships in spatio-temporal domains. They assumed the space to consist of sets of points and used set-theoretic notions to define objects in that space. Their approach is

Table 2
Part of the composition table between two regions with broad boundaries. The relations between R_1 and all the 44 relations is shown



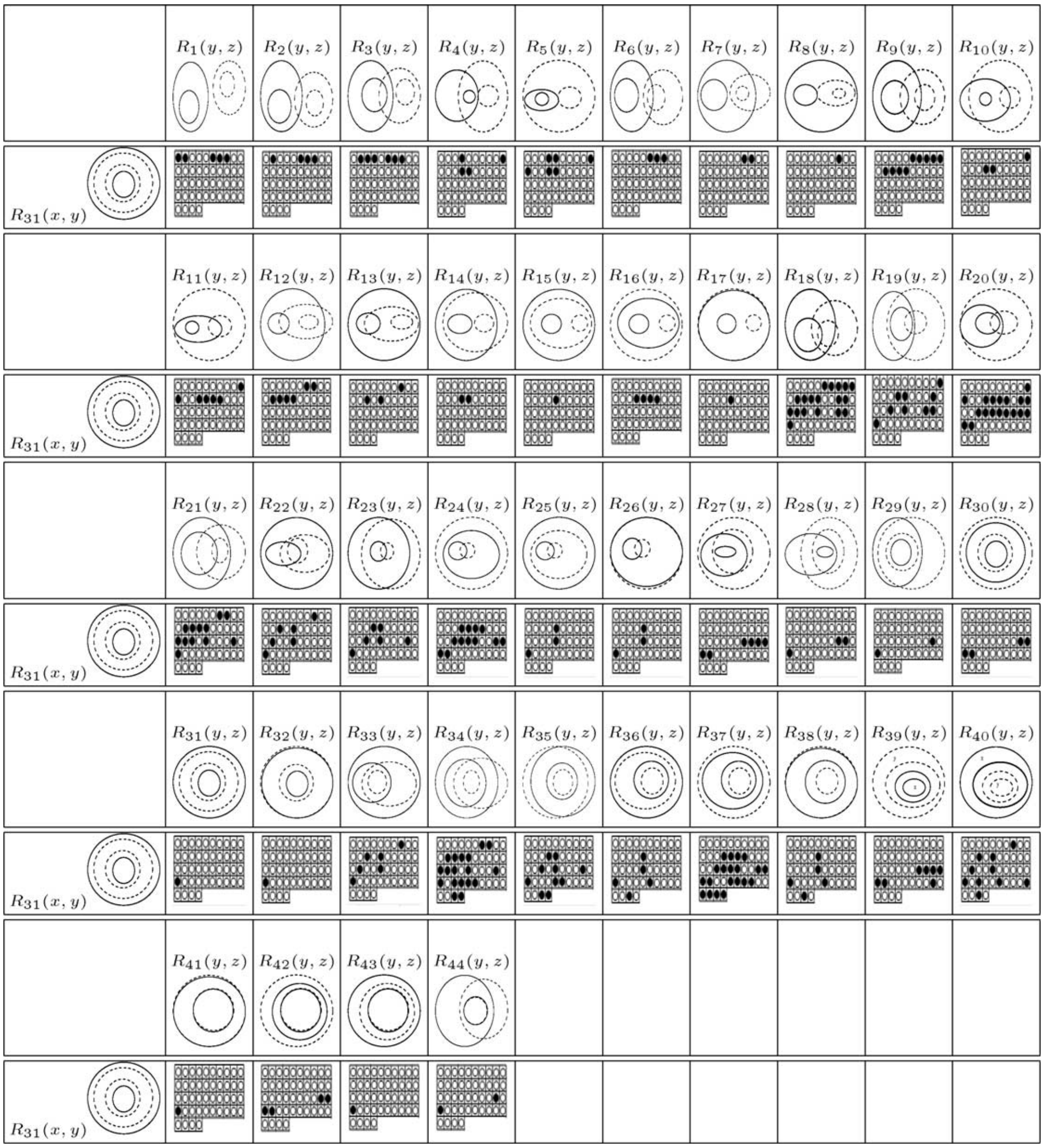
distinctive from the above where space is considered to be discrete, not continuous. The method was used to define a general 'object' and quantitative identifiers are used to qualify the object properties. Extending the method for distinguishing between different types of regions was not proposed.

Vague regions or regions with undetermined boundaries were studied in Refs. [4,7,12,18,20,27]. Only simple convex regions with no holes were considered and the undetermined boundary was represented by a surrounding ring [7,12]. Approaches to spatial reasoning in the literature can generally be classified into (a) approaches using *transitive propagation* and (b) approaches using *theorem proving*.

- *Transitive propagation*. In this approach the transitive property of some spatial relations is utilized to carry out the required reasoning. This applies to the *order relations*, such as `before`, `after` and $(<, =, >)$ (for example, $a < b \wedge b < c \rightarrow a < c$), and to subset relations such as `contain` and `inside` (for example, $\text{inside}(A,B) \wedge \text{inside}(B,C) \rightarrow \text{inside}(A,C)$ and $\text{east}(A,B) \wedge (B,C) \rightarrow \text{east}(A,C)$).

Transitive property of the subset relations was employed by Egenhofer [14] for reasoning over topological relationships. Transitive property of the order relations has been utilised by Mukerjee and Joe [22], Guesgen [19], Lee and Hsu [21] and Papadias and Sellis [24].

Table 3
Part of the composition table between two regions with broad boundaries



The relations between R_{31} and all the 44 relations is shown. *Note*. Object x contains object y in the figure.

Although order relations can be utilised in reasoning over point-shaped objects, they cannot be directly applied when the actual object shapes and proximity of objects are considered.

- *Theorem proving (elimination)*. Here, reasoning is carried out by checking every relation in the full set of *sound* relations in the domain to see whether it is a valid consequence of the composition considered (theorems to be proved) and eliminating the ones which are not consistent with the composition [13].

  Bennett [2] have proposed a propositional calculus for the derivation of the composition of topological relations between simple regions using this method. However, checking each relation in the composition table to prove or eliminate is not possible in general cases and is considered a challenge for theorem provers [5,26].

In general limitation of the methods in the above two approaches can be summarised as follows.

- Spatial reasoning is studied only between objects of similar types, e.g. between two lines or two simple areas. Spatial relations exist between objects of any type and it

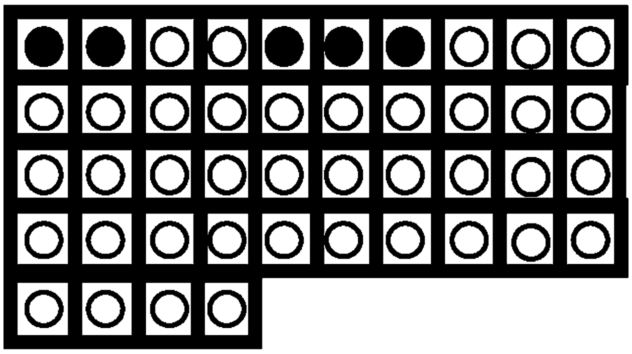


Fig. 11. Notation for representing the result of the composition of relations in Tables 2 and 3.

is limiting to consider the composition of only specific object shapes.

- Spatial reasoning was carried out only between objects with the same dimension as the space they are embedded in, e.g. between two lines in 1D, between two regions in 2D, etc.
- Spatial reasoning is studied mainly between simple object shapes or objects with controlled complexity, for example, regions with holes treated as concentric simple regions. None of the methods in the literature have been presented for spatial reasoning between objects with arbitrary complexity.

The method proposed here is simple and general - only two rules are used to derive composition between objects of arbitrary complexity and is applicable to different types of spatial relations.

5. Conclusions

In this paper, a general approach to qualitative representation and reasoning has been presented. The method is simple and is based on a uniform representation of objects and spatial relationships. Objects are decomposed into representative components and their topology described in an adjacency matrix. The set of sound topological relations between objects are represented by the interaction of the object components. The approach is general where composition of spatial relations can be applied between objects of arbitrary dimension and complexity. An implementation of the method is also presented to demonstrate its validity and generality. Using the reasoning engine, SPARQS, several new composition tables were built between common spatial object types, viz., points, lines, polygons, concave polygons and regions with holes. The engine also includes a more flexible interface where manual input of adjacency and intersection matrices can be used to derive the composition of other arbitrary object shapes. The automatic derivation of composition tables presents an important step towards the realisation of a general qualitative reasoning engine which can be utilised in large spatial databases.

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