

Spatial Reasoning with Place Information on the Semantic Web

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Geographical referencing of data and resources on the Web has become prevalent. Discovering and linking this information poses eminent research challenges to the geospatial semantic web, with regards to the representation and manipulation of information on geographic places. Towards addressing these challenges, this work explores the potential of the current semantic web languages and tools. In particular, an integrated logical framework of rules and ontologies, using current W3C standards, is assessed for modeling geospatial ontologies of place and for encoding both symbolic and geometric references to place locations. Spatial reasoning is incorporated in the framework to facilitate the deduction of implicit spatial relations and for expressing spatial integrity constraints. The logical framework is extended with geo-computation engines that offer more effective manipulation of geometric information. Example data sets mined from web resources are used to demonstrate and evaluate the framework, offering insights to its potentials and limitations.

Keywords: Geospatial semantic web; spatial representation; spatial reasoning.

1. Introduction

Geographic referencing and linking of resources is becoming a commonplace activity on the Web, facilitated by simple mapping applications, as well as the collaborative documentation of place information and creation of web gazetteers by users. Prominent examples of these application include *GeoNames*,^a *Openstreetmap*^b and *DBpedia*.^c The representation of place location differs across resources, ranging from

^ahttp://www.geonames.org/ ^bhttp://www.openstreetmap.org/ ^chttp://www.dbpedia.org/

references to latitude and longitude of a representative point to detailed representation of boundary geometry, as well as, qualitative reference to relative location. For example, place description in web pages are mostly qualitative reflecting the natural way of communicating place information by humans; a place may be described as being in another place or north of another, etc.

Both types of qualitative and quantitative location information are useful for reasoning with and retrieval of geo-referenced information on the Web. However, this information are also associated with a number of problems that can hinder its effective use. The flexible nature of data provision on the Web can result in data being, (a) incomplete, where it is common for resources to include only partial location and identification information for some geographic places, for example, the coverage of place data in Openstreetmap is dense in some areas and sparse in others, and (b) inconsistent, where location information or place names may differ across resources, resulting in possibly multiple different references to the same place entity.

This paper studies the question of whether the current semantic web technologies can be "spatially-enabled" to support the representation and joint manipulation of both qualitative and quantitative location references of place entities.

A framework for the integration of web ontology and rule languages with geospatial information processing is proposed to allow, on the one hand, the encoding of and reasoning over qualitative references to place information, and on the other hand the effective manipulation of the geometric components of this information. The challenges of incorporating spatial logics within the semantic web framework are discussed and possible solutions presented. Applications of the framework are described for the deduction of implicit spatial relationships in web geographic resources and for checking the spatial consistency of these resources.

1.1. Related work

The expressiveness of Web ontology language OWL makes it a suitable modeling platform for different domains. However, it still falls short of supporting more complex domains, such as the geospatial domains, for several reasons: (a) "Triangular knowledge" is not representable in OWL-DL.¹ In particular, complex property compositions which are inference patterns of the form, $\forall x, y, c : R_1(x, y) \land R_2(y, c) \rightarrow R_3(x, c)$ where R_1, R_2 and R_3 are different relations, can not be handled. OWL2 adds a restricted complex property inclusion axiom that can capture a limited form of an inference rule as follows: $R(x, y) \land S(y, c) \rightarrow S(x, c)$ or $R(x, y) \land S(y, c) \rightarrow R(x, c)$. Such axioms only permit the conclusion of a property used in the body of the composition, guaranteeing decidability, but will still not handle the more general form of complex property compositions. (b) To enable qualitative spatial reasoning, a DL language must also support an unrestricted form of role inclusion axioms, namely, $SoT \sqsubseteq R_1 \sqcup \cdots \sqcup R_n$ (o stands for the composition of roles).² Complex role inclusion axioms are not suitable for the expression of procedural implementation of spatial operations,

nor do they offer efficient storage structures or spatial indexing methods, typically offered in spatial database and GIS.

Research efforts have been proposed to extend OWL's underlying description logic to address the requirements for spatial representation and reasoning. However, these mostly lead to undecidability issues and have so far been limited in application. One approach is to extend the ontology language with spatial logics. In Ref. 2, Wessel *et al.* identified spatial reasoning as a key component of DL inference and suggested the need to overcome the limitations of existing DL-based languages to handle composition-based role inclusion axioms (complex property composition) to allow for the possibility of capturing spatial composition inference patterns in the RCC8 composition table. However, such an expressive DL ($\mathcal{ALC}_{RA\Theta}$) was noted to be undecidable.³

A DL \mathcal{ALCI}_{rcc} that only includes role axioms derived from the RCC8 composition table is presented in Ref. 4 to investigate the problem of concept satisfiability using spatial reasoning. The axiomatization of the RCC8 composition table is applied to check the satisfiability of individuals (RSAT) with respect to role box axioms. Role disjointness was added in order to capture the exclusive nature of spatial roles (the 8 JEPD spatial relationships). Moreover, the need for the DL to handle inverse roles was noted to capture converse relational inferences^d to complete the RCC8 spatial network. However, $ALCI_{RCC8}$ was proved undecidable in Ref. 5.

In Ref. 6 a hybrid framework that links a DL reasoner with a GIS was proposed and demonstrated in Ref. 7] with the RacerPro system. The need for representing the geometric component of the geographic information is acknowledged. In Refs. 8 and 9 experiments are presented to link DL and RCC spatial logic in a hybrid framework that links an RCCBox and a spatial reasoning engine with a TBox and an ABox. Qualitative spatial reasoning is carried out on the ABox and new deductions can be asserted, if they do not violate the spatial consistency of the knowledge base. The DL engine and spatial reasoning engine are not connected and the work assumes an ABox populated with pre-computed spatial relationships representing a complete spatial scene. Similar to the above approaches, our work assumes both geometric and qualitative location references, but allows for incompleteness of the underlying geographic scenes, where geometric location information may only exist for some place entities in the scene, and thus the need for mixed spatial reasoning is suggested. In addition, the goal is also to overcome the problem of undecidability of the extended DL approach.

Recently the idea of using rule languages as an ontology paradigm, inspired by Description Logic Programs^{10,11} has been adopted in the OWL2 specification. In particular, OWL2 RL is a profile aimed specifically at supporting reasoning tasks using existing rule-based systems, e.g. Prolog, and includes sufficient constructs to support a wide range of ontologies.¹² It provides an axiomisation, as first-order implications, of the RDF(S)/OWL2 semantics, includes complex property composition

^dHence the \mathcal{I} in the DL \mathcal{ALCI} .

axioms and can be used in existing logic programming engines which, in comparison with DL reasoners, are generally better at processing queries over large instance bases.

2. Spatial Reasoning Framework

In this paper a framework is proposed that incorporates the OWL2 RL and a spatial database system. A method for embedding qualitative spatial reasoning in a logic programming framework is devised and translated to sets of spatial rules for deduction and integrity checking in the rule and ontology language. Hybrid spatial reasoning is facilitated by linking the reasoning engine with the external spatial database management system, for storing and management of the geometric components of the spatial objects in the ontology.

The architecture of the proposed framework is outlined in Figure 1. The framework combines a qualitative reasoning engine in OWL2 RL with a spatial database management system. The representation of place concepts is split between the OWL2 RL ontology and the spatial database system, with unique links maintained between both components. OWL2 RL is used for developing a qualitative spatial reasoning engine that implements both retrieval and integrity maintenance checking tasks over the place ontology. The qualitative reasoning engine communicates with the external geometry processing functions in the spatial database to evaluate required spatial properties as appropriate. The challenges and solutions adopted to implement the framework in OWL2 RL are described below.

2.1. Qualitative spatial reasoning in OWL2 RL

Supporting spatial reasoning in OWL2 RL involves the representation in the language of topological reasoning rules encoded in the RCC8 composition table and the



Fig. 1. The hybrid spatial reasoning framework.

implementation of the consistency checking algorithm that utilises those rules over a network of topological constraints between spatial objects. In the composition table only a subset (25) of the possible 64 compositions of spatial relations produce definite resulting relation, i.e. only one relation, e.g. $inside(x, y) \wedge inside(y, z) \rightarrow$ inside(x, z), and can be represented as rules of the form $R_1(x, y) \wedge R_2(y, z) \rightarrow$ $R_3(x, z)$. The rest of the compositions cannot be represented as Horn rules in the logic framework employed here. To overcome this problem and ensure the closure of the full set of topological relations, it is proposed to transform the spatial reasoning problem into a representation space that can be handled within the rule ontology language.

A generalised composition table, proposed in Ref. 13, is used here as a basis for this transformation. The table uses a set of 12 general relations, denoted RCC12, that are themselves defined in terms of the eight RCC relations as shown in Figure 2. The composition of the RCC12 relations, shown in Table 1, results in definite spatial relations and thus eliminates the need for expressing disjunctions.

Generalised Relation	Union of RCC8 Base Relations
C(a,b) [Connected]	$a\{PO, TPP, NTPP, EQ, NTPP^{-1}, TPP^{-1}, EC\}b$
DC(a,b) [Disconnected]	a{DC}b
P(a,b) [Part-of]	a{TPP,NTPP,EQ}b
$P^{-1}(a,b) [(Part-of)^{-1}]$	$a\{TPP^{-1}, NTPP^{-1}, EQ\}b$
$coP(a,b) = \neg P(a,b) [\neg Part-of]$	$a\{PO, NTPP^{-1}, TPP^{-1}, EC, DC\}b$
$coP^{-1}(a,b) = \neg P^{-1}(a,b) \ [\neg (Part-of)^{-1}]$	$a\{PO, NTPP, TPP, EC, DC\}b$
O(a,b) [Overlapping]	$a\{PO, TPP, NTPP, EQ, NTPP^{-1}, TPP^{-1}\}b$
DR(a,b) [Discrete From]	$a\{EC, DC\}b$
NTP(a,b) [Non-tangential Part-of]	a{ <i>NTPP</i> }b
$NTP^{-1}(a,b)$ [(Non-tangential Part-of) ⁻¹]	$a\{NTPP^{-1}\}b$
coNTP(a,b) [¬ Non-tangential Part-of]	$a\{PO, TPP, EQ, NTPP^{-1}, TPP^{-1}, EC, DC\}b$
$coNTP^{-1}(a,b) [\neg (Non-tangential Part-of)^{-1}]$	$a\{PO, TPP, EQ, NTPP, TPP^{-1}, EC, DC\}b$



Fig. 2. A set of 12 generalised relations (RCC12) and their corresponding set of RCC8 relations.

				Table 1.	. The l	RCC12 cor	nposition t	able.				
	C	DC	Р	P^{-1}	coP	coP^{-1}	0	DR	NTP	NTP^{-1}	coNTP	$coNTP^{-1}$
C	*	coP	C	×	*	*	*	coNTP	0	*	×	*
DC	coP^{-1}	*	coP^{-1}	DC	*	*	coP^{-1}	*	coP^{-1}	DC	*	×
P	×	DC	P	*	*	coP^{-1}	*	DR	NTP	*	*	$coNTP^{-1}$
P^{-1}	U	coP	0	P^{-1}	coP	*	0	$_{coP}$	0	NTP^{-1}	coNTP	×
coP	*	*	*	coP	*	*	*	*	*	coP	*	*
coP^{-1}	×	*	coP^{-1}	*	*	*	*	*	coP^{-1}	*	×	×
0	*	coP	0	*	*	*	*	coP	0	*	*	*
DR	$coNTP^{-1}$	*	coP^{-1}	DR	*	*	coP^{-1}	*	coP^{-1}	DC	*	×
$_{MTP}$	×	DC	NTP	*	*	coP^{-1}	*	DC	NTP	×	×	coP^{-1}
NTP^{-1}	0	coP	0	NTP^{-1}	coP	*	0	$_{coP}$	0	NTP^{-1}	$_{coP}$	×
coNTP	*	*	*	coNTP	*	*	*	*	*	coP	*	*
$coNTP^{-1}$	*	*	$coNTP^{-1}$	*	*	*	*	*	coP^{-1}	*	*	*

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RCC8 Relation	Conjunction of RCC12 Base Relations
EC	$C \wedge DR$
DC	DC
EQ	$P \wedge P^{-1}$
PO	$O \wedge coP \wedge coP^{-1}$
NTPP	$NTP \wedge coP^{-1}$
TPP	$P \wedge coP^{-1} \wedge coNTP$
$NTPP^{-1}$	$NTP^{-1} \wedge coP$
TPP^{-1}	$P^{-1} \wedge coP \wedge coNTP^{-1}$

Table 2. Mapping RCC8 to RCC12 relations.

Reasoning with RCC12 is suitable for the proposed Horn rule framework for the following reasons.

- (1) Compositional inferences in the RCC12 composition table can be captured natively within a Horn framework. Entries in the RCC12 table are all definite relations and disjunction of RCC8 table entries correspond to intersection of sets of RCC12 relations.
- (2) Computing the closure of a set of relational constraints using RCC8 reasoning generates the same compositional inferences (the same refined set of relational constraints of RCC8 base relations) as the closure of the same set of relational constraints using the RCC12 composition table.¹³
- (3) All 12 generalised relations are in the maximal tractable set \mathcal{H}_8 ,¹⁴ that is closed under intersection and thus the intersection of any base RCC12 relation is also in the set \mathcal{H}_8 . Hence, providing a mapping exists between a set of RCC8 relations and a corresponding conjunctive set of generalised base relations, deciding path consistency over the resultant generalised relational constraints, is sufficient for deciding global consistency of the set of relational constraints.

To use the RCC12 as a base for the spatial reasoning system, four sets of spatial rules need to be defined. These are: deduction rules (\otimes) to represent the inferences in the RCC12 composition table, converse rules (\smile) to represent the converse of the RCC12 relations, intersection rules (\cap) to determine whether relations have a valid intersection (integrity rules) and mapping rules between RCC8 and RCC12 relations.

Mapping Rule Set $(QSR_{map} \rightarrow \text{ and } QSR_{map} \leftarrow)$: To use the RCC12 composition table, all RCC8 constraints need be represented first as conjunctions of RCC12 relations. Hence, a mapping is needed from base RCC8 relations to RCC12 relations. The mappings are defined in Table 2. The reverse mapping from RCC12 to RCC8 $(QSR_{map} \leftarrow)$ need also be defined to represent the results.

Deduction and Converse Rule Sets $(QSR_{RCC12} \otimes \text{ and } QSR_{RCC12} \sim)$: Rules for the deduction of spatial relationships can be derived directly from the RCC12 composition table (Table 1). A set of 60 composition rules representing the definite entries of the RCC12 composition table are defined. Some example rules include the following.

$$P^{-1}(a,b) \wedge C(b,c) \to C(a,c)$$
$$P^{-1}(a,b) \wedge DC(b,c) \to coP(a,c)$$
$$P(a,b) \wedge DC(b,c) \to DC(a,c)$$

Also, an associated set of rules are defined to capture converses of deduction rules.

Integrity Rule Sets $(QSR_{RCC12}\cap)$: To propagate the spatial constraints over a network of spatial relations and check the spatial consistency of the ontology base, rules are needed to simulate the intersection operation over spatial relations. These are represented as spatial integrity rules in the rule base. An integrity rule evaluates not only the relations between region pairs $\langle a, b \rangle$ and $\langle b, c \rangle$, but also the consistency of the relation(s) between $\langle a, c \rangle$. For example, from the composition table we know that $P^{-1}(a,b) \wedge NTP(b,c)$ implies O(a,c). If a relation is defined between a and c that contradicts O(a,c) (that is any relation in the complement set of O), a spatial integrity violation error should be triggered. In particular, the existence of either the relation DR(a,c) or DC(a,c) (both in the complement set of O) will trigger an integrity error. Integrity rules are defined in the engine to capture this scenario as follows.

$$P^{-1}(a,b) \wedge NTP(b,c) \wedge (DR(a,c) \vee DC(a,c)) \rightarrow error(\ldots)$$

A complete set of spatial integrity rules can therefore be derived for all entries of the composition table by identifying the non-intersecting set of relations for *RCC*12.

Hence, the complete set of qualitative reasoning rules (QSR_{RCC12}) consists of the set of composition rules $(QSR_{RCC12^{\otimes}})$, the set of converse rules $(QSR_{RCC12^{\sim}})$, the set of intersection or integrity rules $(QSR_{RCC12^{\cap}})$ and the set of mapping rules $(QSR_{map^{\rightarrow}})$ and $(QSR_{map^{\leftarrow}})$.

2.2. Implementation of the spatial rule engine

The framework has been implemented using the Jena2 Semantic Web toolkit^e and the Oracle spatial database system. In general, there are two kinds of spatial reasoning tasks over a geospatial ontology.

- Derivation of implicit topological relations. This can be achieved using deduction and converse rules — effectively computing the closure of a set of spatial relational constraints.
- (2) Checking the spatial consistency using the full set of deduction, converse and integrity rules.

^ehttp://dsonline.computer.org/0211/f/wp6jena.htm.



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Fig. 3. Sample snapshots of the implemented system and interface. (a) Explicit spatial relations (solid lines) and derived spatial relations (dashed lines), (b) Adding new spatial relationships between regions in the ontology, (c) Resulting topological errors with problem nodes and edges highlighted

The application of qualitative spatial reasoning to evaluate whether the relationship can be automatically deduced from the stored facts is supported by external calls to geometric computation engine to compute the required relationship (using suitable, stored geometric representations of the objects, if available). This strategy is encoded in the definition of the deduction rule sets that resort to calling an external function to compute the required relationship. For example, in the rule $EC(a, b) \wedge EQ(b, c) \rightarrow EC(a, c)$, the conclusion of EC(a, c) would only be inferred if both relations EC(a, c) and EQ(c, b) can be satisfied. The facts can either be derived directly from stored facts in the ontology, or can be inferred by querying the spatial rule sets. If the call is not satisfied using the qualitative reasoning rules, the engine defaults to rules that invoke external calls to the spatial database system to compute the relationships. A set of builtins (spatial operators) are defined to compute the set of 8 base relationships.

Figure 3 depicts snapshots of the system interface, where stored relationships between regions in the ontology are shown. Derived relationships are computed using

Source	Interpreted Relation	Actual Relation
Llanishen: "it covers all of the geographical areas of Llanishen, Birchgrove, and Thornhill".	P^{-1} (Llanishen, Thornhill)	P^{-1}
Llanishen is adjacent to Thornhill , from the Llanishen 3×3 table.	$EC \ (Llanishen, \ Thornhill)$	
Ely , <i>"is bounded by Fairwater, and Gabalfa to the northwest; Caerau, to the south;</i> Culverhouse Cross to the west".	EC (Ely, Culverhouse Cross)	
Culverhouse Cross , "falls within the southwest- ern tip of the Ely , ward".	P (Culverhouse Cross, Ely)	Р
Llanishen "it covers all of the geographical areas of Llanishen, Birchgrove, and Thornhill."	P^{-1} (Llanishen, Birchgrove)	
Llanishen is adjacent to Birchgrove , from the Llanishen 3×3 neighbourhood table	EC (Llanishen, Birchgrove)	EC

Table 3. Topological inconsistencies in wikipedia articles for districts and wards in cardiff.

the deduction rules and spatial inconsistencies are detected after the application of the integrity rules.

The system is also capable of tracing and reporting the rules which triggered the errors to identify the source of the problem.

3. Application and Evaluation

The place ontology is transformed into a rule base and stored as RDF triples in Jena2. To test and evaluate the applicability of the developed system and framework, a place ontology base was built, using sample data for the area of Cardiff, South Wales in the UK, from three representative data sources: GeoNames,^f a data set from the Ordnance Survey,^g representing the official administrative boundaries for Wales and Wikipedia; containing many dedicated pages for place descriptions provided by users and is used as a source for qualitative spatial relationships.

Using a Qualitative Spatial Ontology

A total of just over 200 spatial relationships between 74 distinct wards and districts in the area of Cardiff have been extracted from Wikipedia pages and used to populate an OWL2 RL place ontology.

Applying the spatial integrity rules on this ontology resulted in the detection of 3 inconsistent topological relations, shown in Table 3. The table shows the source extracts from Wikipedia that are in conflict, where the column entitled "Actual" refers to the spatial relation that exist in reality between the regions.

^fGeonames, http://www.geonames.org

^gNational mapping agency of Great Britain.

Wale Wale South Glam Vale o Vale of Glamo (a) (b) Llanhara Radu Rum Ely. Cardiff Barry Sully Rhoose Wale 0 South Glamorgan (c) (d)

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Fig. 4. (a) Spatial relations as derived from Wikipedia. (b) Possible implications for spatial relations between the two regions based on (a) — regions must be connected. (c) Actual location of the region South Glamorgan indicated as a point (marker with letter A) in the sea in GeoNames.
(d) Computed relationship between the two regions based on (c) contradicts implied relations in (b) results in a spatial integrity error.

Manual evaluation of all 200 topological relations in the ontology revealed 16 possible topological inconsistencies. There are 5476 possible topological relations between all 74 Wards and Districts in Wales (n^2 where n is the number of Wards and Districts). Qualitative reasoning with only 3.46% (200) of these relations allowed the detection of 18.75% (3) of all possible errors.

Evaluation with a Hybrid Ontology

The goal of this test is to compare the spatial similarity of different resources that contain a mixed set of qualitative and quantitative spatial data. The following qualitative relations were extracted from Wikipedia for the region of South Glamorgan; an administrative subdivision of Wales, and are diagrammatically depicted in Figure 4(a), $P^{-1}(Wales, Vale-of-Glamorgan)$ and P(Vale-of-Glamorgan, South-Glamorgan).

Spatial composition of the above relations imply the following relationships (using a direct mapping to RCC8) between the regions of Wales and South Glamorgan, as shown in Figure 4(b): $P^{-1}(Wales, Vale-of-Glamorgan) \wedge$

 $P(Vale \circ f - Glamorgan, South - Glamorgan) \rightarrow PO \lor TPP \lor NTPP \lor EQ \lor NTPP^{-1} \lor TPP^{-1}(Wales, South - Glamorgan).$

The following integrity rule also applies, indicating that the two regions cannot be disconnected: $P^{-1}(a, b) \wedge P(b, c) \wedge DC(a, c) \rightarrow error(a, c)$. Firing this integrity rule results in an external call to the spatial database to evaluate whether the two regions are disjoint (DC). The call returns 'True' and an integrity error is triggered. The error stems from the fact that the point location representing the region of South-Glamorgan in the GeoNames data set has been digitised outside the boundary of Wales, as shown in Figure 4(c) and (d) (showing the point in the sea). This fact contradicts with the derived facts from the Wikipedia ontology that South Glamorgan must be connected to Wales. This demonstrates how the engine can be useful for checking the similarity of spatial implications and hence the consistency of information across multiple data resources.

4. Conclusions

As geo-referencing of resources on the web becomes more popular, methods to support the search, sharing and linking of these resources are needed. Qualitative and geometric spatial reasoning are established complementary techniques for the representation and manipulation of spatial information, but in practice they have tended to operate in isolation of each other. Both forms of reasoning are needed for the manipulation of spatial and geo-referenced information on the web. However, supporting these forms of reasoning in web ontology languages is a challenge, where on one hand, the ontology languages are not equipped for the effective or efficient geometric representation and manipulation of location and shape, and on the other hand, composition of spatial relations for qualitative spatial reasoning requires the support of complex property composition axioms.

In this paper, we address these problems by a framework that links the OWL2 RL rule ontology language with a spatial database system and shares the representation and reasoning tasks between both systems. Supporting spatial reasoning in OWL2 RL involved first, using a generalised set of topological relations that allowed the production of a definite composition table, and hence overcoming the problem of representing disjunctive compositional inferences, and second, the implementation of a path-consistency algorithm to solve spatial constraint networks by constructing spatial rule sets for the composition and intersection of those constraints. The strength of the method lies in the fact that the transformation preserves the tractability of the spatial reasoning problem and can thus be realised directly in a rule-based engine. Several research questions still need to be explored, including, finding optimal strategies for structuring spatial objects and relationships to enhance the effectiveness of both modes of reasoning, and their possible utilisation in the GeoSPARQL semantic web query language.^h

 $^{\rm h}$ http://www.opengeospatial.org/standards/geosparql

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