Topological Representation and Reasoning in Space

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Abstract

In this paper, an approach is presented for the representation and reasoning over qualitative spatial relations. A set-theoretic approach is used for representing the topology of objects and underlying space by retaining connectivity relationships between objects and space components in a structure, denoted, adjacency matrix. Spatial relations are represented by the intersection of components, and spatial reasoning is achieved by the application of general rules for the propagation of the intersection constraints between those components. The representation approach is general and can be adapted for different space resolutions and granularities of relations. The reasoning mechanism is simple and the spatial compositions are achieved in a finite definite number of steps, controlled by the complexity needed in the representation of objects and the granularity of the spatial relations required. The application of the method is presented over geometric structures which takes into account qualitative surface height information. It is also shown how directional relationships can be used in a hybrid approach for more richer composition scenarios. The main advantages of this work is that it offers a unified platform for handling different relations in the qualitative space which is a step towards developing general spatial reasoning engines for large spatial databases.

KeywordS: Qualitative spatial relations, qualtiative reasoning, topological relationships, orientation relationships, spatial databases

1 Introduction

Representing and manipulating spatial or geometric relations are of primary importance in many application areas of large spatial databases such as, Computer Aided Design, Manufacture and Process Planning (CAD/CAM/CAPP), Geographic Information Systems (GIS) and medical and biological databases. As a result Spatial Reasoning (SR) find application in diverse areas such as Assembly Planning, Robotics, Constraint Driven Design and Drafting and Machine Selection and Specification. GIS are based on a range of spatial reasoning techniques for manipulating geographic features on one or more data layers, such as in processing spatial join queries, where sets of geographically referenced features are overlaid in the search for regions satisfying particular constraints. Such application domains are characterised by handling very large sets of entities, relationships and constraints and their manipulation usually involve substantial computational costs.

Qualitative Spatial Representation and Reasoning (QSRR) techniques are being developed to complement the traditional quantitative methods in those domains. Many typical problems could benefit from qualitative manipulation when precise geometric information are neither available nor needed. Applications of QSRR include, qualitative spatial scene specification and scene feasibility problems, checking the similarity and consistency of data sets, integrating different spatial sets [EGA98], and in initial pruning of search spaces in spatial query processing. Research is also ongoing for incorporating QSRR in the definition and implementation of spatial query languages. However, the qualitative approach has obvious limitations where useful characteristics of spatial objects such as shape and size are not used. Also, its application becomes limited when exact positions and tolerance constraints are considered. Hence, it can be argued that both the quantitative and qualitative approaches have complementary areas of strength and that any system which can combine the two paradigms in a way which uses their strength would be an effective platform for a range of novel and conventional applications.

This paper presents an approach to the representation of and reasoning over qualitative spatial relations, namely topological and orientation. It is shown how a representation strategy for storing connectivity relationships between objects and space components forms the basis of a reasoning mechanism for the composition of spatial relationships. The two types of relationships are treated individually and then combined to demonstrate the effectiveness of a hybrid approach. Examples are given to illustrate the applicability of the reasoning process on objects of arbitrary complexity and dimension. The extension of formalism to higher dimension spaces is readily recognised.

The paper is structured as follows. In section 2, an overview is given of the different types of qualitative relationships in space. Section 3 outlines the representation approach in topological spaces. In section 4, the reasoning method is presented and applied over topological relations between non-simple objects. Representation and reasoning over orientation spaces is given in section 5 and the combined spaces are treated in section 6. Discussions and conclusions are given in section 7 where it is also shown how semi-quantitative shape information could be incorporated in the formalism.

The Qualitative Frame of Reference

Different types of qualitative spatial relations can be presented on a qualitative frame of reference in an analogy to the quantitative (absolute) frame of reference. One of the objects is used to represent the 'origin' with respect to which the other object is referenced. An object in space possess three degrees of freedom which determine its spatial relationship with other objects, namely, transition, rotation and scaling (enlargements or shrinking). Accordingly, three main axes of variation can be established as shown in figure 1, namely,

1. Topology-proximity axis *(P-axis)*: over which the variation represents relationships resulting from the relative transition of objects.

2. Orientation axis *(O-axis)*: over which the variation represents relationships resulting from the relative rotation of objects.

3. Size axis *(S-axis)*: where scaling variations are represented.

Two types of rotations can be distinguished on the orientation axis: the rotation of the object around the reference object and the rotation of the object around itself. Hence, there is a need to define orientation relations from the points of view of each object. This type of orientation is called body orientation and is using an intrinsic frame of reference[RS88]. On the other hand, *Extrinsic* orientations are when a fixed external frame of reference is used for both the object spaces, for example cardinal direction orientation (east, west, north, south)

Inter-dependencies of the axes can be recognised as follow,

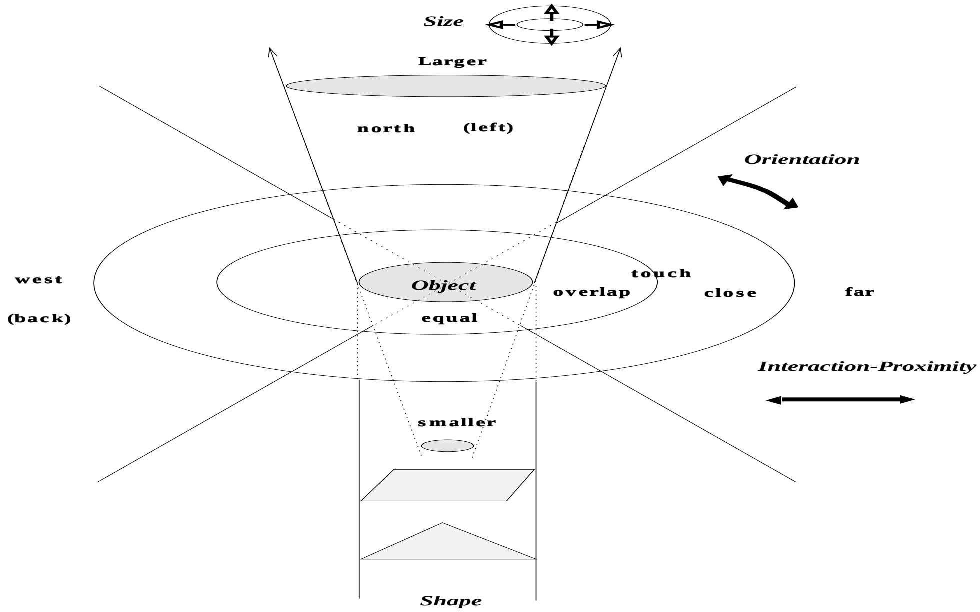


Figure 1: Qualitative frame of reference for spatial relationships.

1. Relationships along the Size axis are independent of the other two axes.

2. Relationships along the Topology-Proximity axis could depend on the size of the objects involved in the case where the objects are in close proximity. For example, when objects are very close, changing the size of the objects can transform a relationship of `disjoint` into `overlap` or the relationship of `equal` to `contain` or `inside`.

3. Relationships on the Orientation axis could be affected by both size and proximity of the objects involved. An object `front` of another can become also `left` or `right` of it, if it increases in size or gets closer to the other object.

Shape is another aspect of the qualitative representation. However, it cannot be represented by linear ordered continuous variations. It is added on the figure for illustrative purposes. Accordingly an object O_1 can be qualitatively described with reference to another object O_2 by a triple (P, O, S) which is the qualitative equivalence to the (X, Y, Z) or (R, θ, ϕ) in a quantitative frame of reference.

2 The Formalism

This section first addresses the problem of qualitative representation of objects with random spatial complexity and their topological relationships. The reasoning formalism is then presented, consisting of

a) general constraints to govern the spatial relationships between objects in space, and b) general rules to propagate relationships between those objects. Both the constraints and the rules are based on a uniform representation of the topology of the objects, their embedding space and the representation of the relationships between them.

2.1 The General Representation Approach

Objects of interest and their embedding space are divided into components according to a required resolution. The connectivity of those components is explicitly represented. Spatial relations are represented by the intersection of object components [AW94] in a similar fashion to that described in [Ege94] but with no restriction on object components to consist only of two parts (boundary and interior).

2.1.1 The Underlying Representation of Object Topology

Let S be the space in which the object is embedded. The object and its embedding space are assumed to be *dense* and *connected*. The embedding space is also assumed to be infinite. The object and its embedding space are decomposed into components which reflects the objects and space topology such that,

1. No overlap exists between any of the representative components.

2. The union of the components is equal to the embedding space.

The topology of the object and the embedding space can then be described by a matrix whose elements represent the connectivity relations between its components. This matrix shall be denoted *adjacency matrix*. In the decomposition strategy, the complement of the object in question shall be considered to be infinite, and the suffix 0, e.g. (x_0) is used to represent this component. Hence, the topology of a space S containing an object x is defined using the following equation.

$$x = \bigcup_{i=1}^{n} x_i \qquad (1)$$

$$S_x = x \cup x_0 \qquad (2)$$

where S_x is used to denote the space associated with object x.

In figure 2 different possible decompositions of a simple convex polygon and its embedding space are shown along with their adjacency matrices. In 2 (a), the object is represented by two components, a

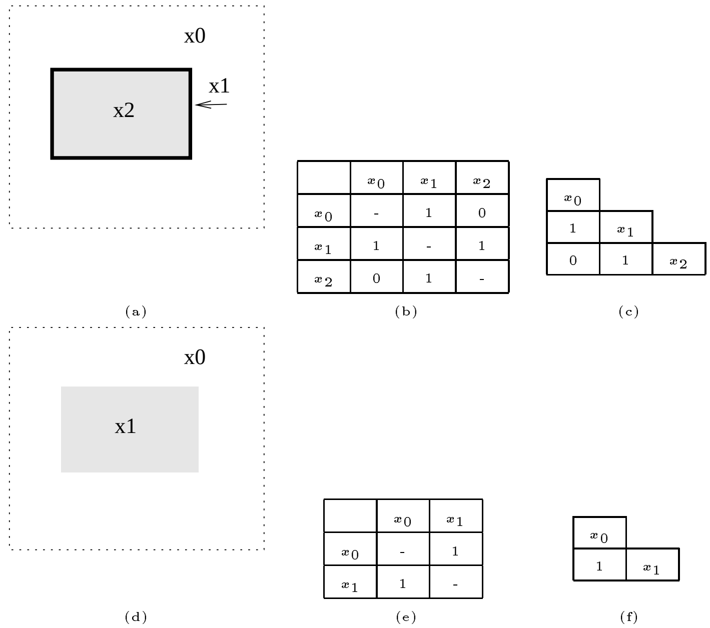


Figure 2: (a), (d) Possible decompositions of a simple convex polygon and its embedding space. (b), (e) Adjacency matrices corresponding to the two shapes in (a) and (d) respectively. (c), (f) Half the symmetric adjacency matrix is sufficient to capture the object representation.

linear component x_1 and an areal component x_2 and the rest of the space is represented by an infinite areal component x_0 representing the surrounding area. In 2(d), only one areal component is used to represent the polygon. Both representations are valid and may be used in different contexts. Different decomposition strategies for the objects and their embedding spaces can be used according to the precision of the relations required and the specific application considered. The higher the resolution used (or the finer the components of the space and the objects), the higher the precision of the resulting set of relations in the domain considered.

The fact that two components are connected is represented by a (1) in the adjacency matrix and by a (0) otherwise. Since connectivity is a symmetric relation, the resulting matrix will be symmetric around the diagonal. Hence, only half the matrix is sufficient for the representation of the object's topology and the matrix can be collapsed to the structure in figure 2(c) and (f).

Semi-bounded areas of the embedding space can also be represented (as virtual components) if needed. For example, figure 3(a) shows a possible decomposition of a concave shaped object and its embedding space. In 3(b) the adjacency matrix for its components is presented. The object is represented by two

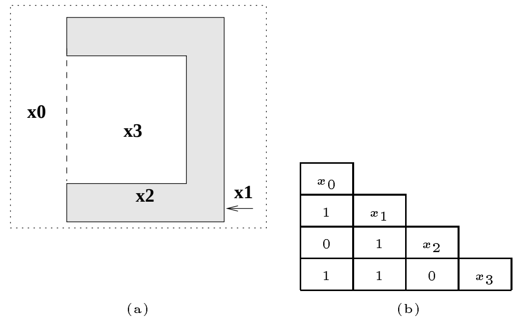


Figure 3: (a) Using virtual components to represent semi-bounded components (of interest) in space. (b) Adjacency matrix for the shape in (a).

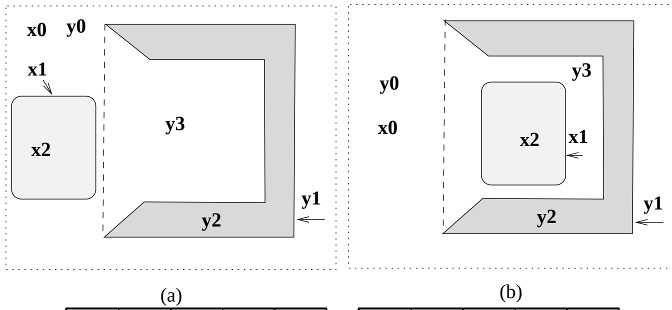
components a linear component x_1 and an areal component x_2 and the rest of its embedding space is represented by a finite areal component x_3 (representing the virtual enclosure) and infinite areal component x_0 representing the surrounding area.

2.1.2 The Underlying Representation of Spatial Relations

In this section, the representation of the topological relations through the intersection of their components [ECP94, EH90] is adopted and generalised for objects of arbitrary complexity. Distinction of topological relations is dependent on the strategy used in the decomposition of the objects and their related spaces. For example, in figure 4 different relationships between two objects x and y are shown, where in 4(a) the x is outside y and in 4(b) x is inside y. Object y is decomposed into two components y_1 and y_2 and the rest of the space associated with y is decomposed into two components: y_3 representing the enclosure and y_0 representing the rest of the space. Note that it is the identification of the (virtual) component y_3 that makes the distinction between the two relationships in the figure. The complete set of spatial relationships are identified by combinatorial intersection of the components of one space with those of the other space.

If $R(x,y)$ is a relation of interest between objects x and y, and X and Y are the spaces associated with the objects respectively such that m is the number of components in X and l is the number of components in Y, then a spatial relation $R(x,y)$ can be represented by one instance of the following equation:

$$R(x,y) \quad = \quad X \cap Y$$



	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	1	0	0	0
x_2	1	0	0	0

(c)

	y_0	y_1	y_2	y_3
x_0	1	1	1	1
x_1	0	0	0	1
x_2	0	0	0	1

(d)

Figure 4: Different qualitative spatial relationships can be distinguished by identifying the appropriate components of the objects and the space. (c) and (d) corresponding intersection matrices.

$$
\begin{aligned}
&= \left(\bigcup_{i=1}^{m} x_i\right) \cap \left(\bigcup_{j=1}^{l} y_j\right) \\
&= (x_1 \cap y_1, \cdots, x_1 \cap y_l, x_2 \cap y_1, \cdots, x_m \cap y_l)
\end{aligned}
$$

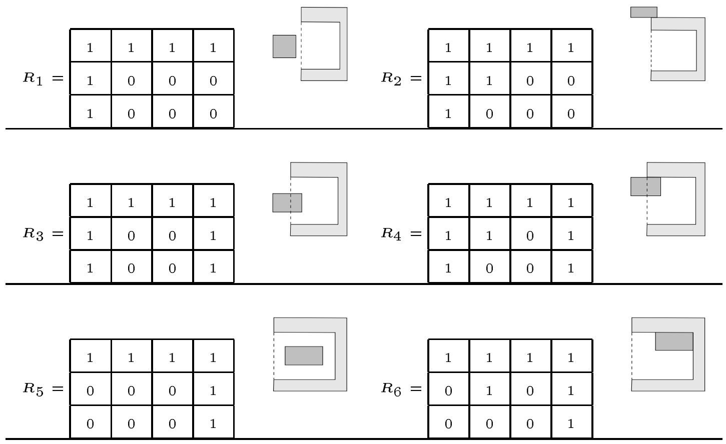
The intersection $x_i \cap y_j$ can be an empty or a non-empty intersection. The above set of intersections shall be represented by an intersection matrix, as follows,

$R(x, y) =$

	y_0	y_1	y_2	$\cdots$
x_0				
x_1				
x_2				
$\vdots$				

For example, consider the intersection matrices in figure 4(c) and (d). The components x_1 and x_2 have a non-empty intersection with y_0 in 4(c) and with y_3 in 4(d).

Different combinations in the intersection matrix can represent different qualitative relations. The set of valid or sound spatial relationships between objects is dependent on the particular domain studied. Also, properties of the objects would affect the set of possible spatial relationships that can exist between them. For example, if one of the objects is solid and the other is permeable, there cannot be any intersection of the inside of the solid object with any other component of the other object. Objects of



$$R_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad R_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad R_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

$$R_5 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_6 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 5: A set of 6 spatial relationships between two solid bodies. The decomposition of objects are as in figure 4.

different size or shape cannot be involved in certain spatial relations such as `equal` or `contain` between the smaller and the larger object.

The example in figure 5 demonstrates the six possible spatial relations that can exist between two solid objects; a simple convex polygon and a simple concave one, along with their intersection matrices. The example can be used to represent many situations, for example, putting a box into a container, carving a hole in a solid shape by a tool, etc. Note that since object y is a solid object, the component y_2 will always have only one intersection relation with x_0.

2.2 The General Reasoning Formalism

The reasoning approach consists of: a) general constraints to govern the spatial relationships between objects in space, and b) general rules to propagate relationships between the objects.

2.2.1 General Constraints

The intersection matrix is in fact a set of constraints whose values identifies specific spatial relationships. The process of spatial reasoning can be defined as the process of propagating the constraints of two spatial relations (for example, $R_1(A, B)$ and $R_2(B, C)$), to derive a new set of constraints between objects. The derived constraints can then be mapped to a specific spatial relation (i.e. the relation $R_3(A, C)$).

A subset of the set of constraints defining all possible spatial relations are general and applicable to any relationship between any objects. These general constraints are a consequence of the initial assumptions used in the definition of the object and space topology. The two general constraints are:

1. Every unbounded (infinite) component of one space must intersect with at least one unbounded (infinite) component of the other space.

2. Every component from one space must intersect with at least one component from the other space.

2.2.2 General Reasoning Rules

Composition of spatial relations is the process through which the possible relationship(s) between two object x and z is derived given two relationships: R_1 between x and y and R_2 between y and z. Two general reasoning rules for the propagation of intersection constraints are presented. The rules are characterised by the ability to reason over spatial relationships between objects of arbitrary complexity in any space dimension. These rules allow for the automatic derivation of the composition (transitivity) tables between any spatial shapes [AA95, RCC92b].

Reasoning Rules

Composition of spatial relations using the *intersection* representation approach is based on the transitive property of the subset relations. In what follows the following subset notation is used. If x' is a set of components (set of point-sets) $\{x_1, \cdots, x_{m'}\}$ in a space X, and y_j is a component in space Y, then $\sqsubseteq$ denotes the following subset relationship.

- $y_j \sqsubseteq x'$ denotes the subset relationship such that: $\forall x_i \in x'(y_j \cap x_i \neq \phi) \wedge y_j \cap (X - x_1 - x_2 \cdots - x_m) = \phi \ where \ i = 1, \cdots m'$. Intuitively, this symbol indicates that the component y_j intersects with every set in the collection x' and does not intersect with any set outside of x'.

If x_i, y_j and z_k are components of objects x, y and z respectively, then if there is a non-empty intersection between x_i and y_j, and y_j is a subset of z_k, then it can be concluded that there is also a non-empty intersection between x_i and z_k.

$$(x_i \cap y_j \neq \phi) \ \wedge \ (y_j \subseteq z_k) \rightarrow (x_i \cap z_k \neq \phi)$$

This relation can be generalised in the following two rules. The rules describe the propagation of intersections between the components of objects and their related spaces involved in the spatial composition.

Rule 1: Propagation of Non-Empty Intersections

Let $x' = \{x_1, x_2, \cdots, x_{m'}\}$ *be a subset of the set of components of space* X *whose total number of components is* m *and* $m' \leq m$; $x' \subseteq X$. *Let* $z' = \{z_1, z_2, \cdots, z_{n'}\}$ *be a subset of the set of components of space* Z *whose total number of components is* n *and* $n' \leq n$; $z' \subseteq Z$. *If* y_j *is a component of space* Y, *the following is a governing rule of interaction for the three spaces* X, Y *and* Z.

$$
\begin{aligned}
(x' \sqsupseteq y_j) \quad \wedge \quad & (y_j \sqsubseteq z') \\
\rightarrow \quad & (x' \cap z' \neq \phi) \\
\equiv \quad & (x_1 \cap z_1 \neq \phi \vee \cdots \vee x_1 \cap z_{n'} \neq \phi) \\
& \wedge (x_2 \cap z_1 \neq \phi \vee \cdots \vee x_2 \cap z_{n'} \neq \phi) \\
& \wedge \cdots \\
& \wedge (x_{m'} \cap z_1 \neq \phi \vee \cdots \vee x_{m'} \cap z_{n'} \neq \phi)
\end{aligned}
$$

The above rule states that if the component y_j in space Y has a positive intersection with every component from the sets x' and z', then each component of the set x' must intersect with at least one component of the set z' and vice versa.

The constraint $x_i \cap z_1 \neq \phi \ \vee \ x_i \cap z_2 \neq \phi \cdots \ \vee \ x_i \cap z_{n'} \neq \phi$ can be expressed in the intersection matrix by a label, for example the label a_r ($r = 1$ or 2) in the following matrix indicates $x_1 \cap (z_2 \cup z_4) \neq \phi$ (x_1 has a positive intersection with z_2, or with z_4 or with both). A $-$ in the matrix indicates that the intersection is either positive or negative.

	z_1	z_2	z_3	z_4	$\cdots$	z_n
x_1	$-$	a_1	$-$	a_2	$-$	$-$

Rule 1 represents the propagation of non-empty intersections of components in space. A different version of the rule for the propagation of empty intersections can be stated as follows.

Rule 2: Propagation of Empty Intersections

Let $z' = \{z_1, z_2, \cdots, z_{n'}\}$ *be a subset of the set of components of space* Z *whose total number of components is* n *and* $n' < n$; $z' \subset Z$. *Let* $y' = \{y_1, y_2, \cdots, y_{l'}\}$ *be a subset of the set of components of* *space Y whose total number of components is l and $l' < l$; $y' \subset Y$. Let x_i be a component of the space* X. Then the following is a governing rule for the spaces X, Y and Z.

$$(x_i \sqsubseteq y') \quad \wedge \quad (y' \sqsubseteq z')$$
$$\rightarrow \quad (x_i \cap (Z - z_1 - z_2 \cdots - z_{n'}) = \phi)$$

Remark: if $n' = n$, i.e. x_i may intersect with every element in Z, or if $m' = m$, i.e. z_k may intersect with every element in X, or if $l' = l$, i.e. x_i (or z_k) may intersect with every element in Y, then no empty intersections can be propagated for x_i or z_k. Rules 1 and 2 are the two general rules for propagating empty and non-empty intersections of components of spaces.

Note that in both rules the intermediate object (y) and its space components plays the main role in the propagation of intersections. The first rule is applied a number of times equal to the number of components of the space of the intermediate object. Hence, the composition of spatial relations becomes a tractable problem which can be performed in a defined limited number of steps.

2.3 Example of Spatial Reasoning with Complex Objects

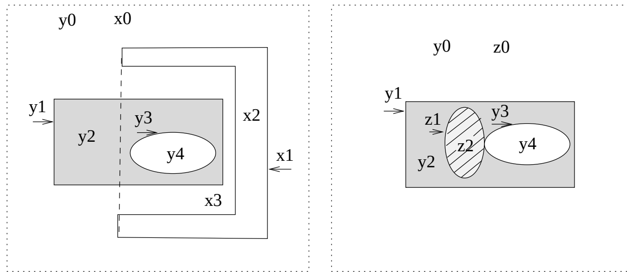
The example in figure 6 is used for demonstrating the composition of relations using non-simple spatial objects. Figure 6(a) shows the relationship between a concave polygon x and a polygon with a hole y and 6(b) shows the relationship between object y and a simple polygon z where z touches the the hole in y. The intersection matrices corresponding to the two relationships are also shown.

Given that the possible set of relationships that can occur between x and z in a certain domain are as shown in figure 5, it is required to derive the possible relationships between these two objects given the situation in figure 6.

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- y_0 intersections:

$$\{x_0, x_1, x_2, x_3\} \sqsupseteq y_0 \quad \wedge \quad y_0 \sqsubseteq \{z_0\}$$
$$\rightarrow \quad x_0 \cap z_0 \neq \phi \wedge x_1 \cap z_0 \neq \phi$$
$$\wedge \quad x_2 \cap z_0 \neq \phi \wedge x_3 \cap z_0 \neq \phi$$



(a) (b)

	y_0	y_1	y_2	y_3	y_4
x_0	1	1	1	0	0
x_1	1	0	0	0	0
x_2	1	0	0	0	0
x_3	1	1	1	1	1

(c)

	z_0	z_1	z_2
y_0	1	0	0
y_1	1	0	0
y_2	1	1	1
y_3	1	1	0
y_4	1	0	0

(d)

Figure 6: (a) and (b) Spatial relationships between non-simple objects x, y and z. (c) and (d) Corresponding intersection matrices.

- y_1 intersections:

$$\{x_0, x_3\} \sqsupseteq y_1 \wedge y_1 \sqsubseteq \{z_0\} \rightarrow x_1 \cap z_0 \neq \phi \wedge x_3 \cap z_0 \neq \phi$$

- y_2 intersections:

$$\begin{aligned} \{x_0, x_3\} \sqsupseteq y_2 \quad & \wedge \quad y_2 \sqsubseteq \{z_0, z_1, z_2\} \\ & \rightarrow \quad x_0 \cap (z_0 \cup z_1 \cup z_2) \neq \phi \\ & \wedge \quad x_3 \cap (z_0 \cup z_1 \cup z_2) \neq \phi \end{aligned}$$

- y_3 intersections:

$$\begin{aligned} \{x_3\} \sqsupseteq y_3 \quad & \wedge \quad y_3 \sqsubseteq \{z_0, z_1\} \\ & \rightarrow \quad x_3 \cap z_0 \neq \phi \wedge x_3 \cap z_1 \neq \phi \end{aligned}$$

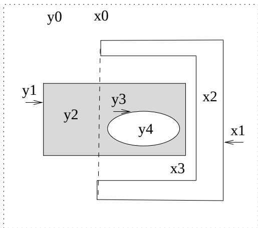
- y_4 intersections:

$$\{x_3\} \sqsupseteq y_4 \ \wedge \ y_4 \sqsubseteq \{z_0\} \quad \rightarrow \quad x_3 \cap z_0 \neq \phi$$

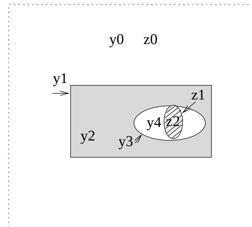
Applying rule 2 we get the following,

- $x_0 \sqsubseteq \{y_0, y_1, y_2\} \ \wedge \ \{y_0, y_1, y_2\} \sqsubseteq \{z_0, z_1, z_2\}$

  x_0 has no empty intersections with components in Z.

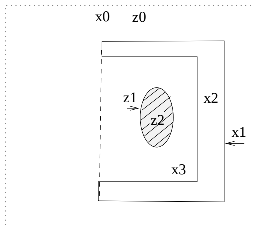


	y_0	y_1	y_2	y_3	y_4
x_0	1	1	1	0	0
x_1	1	0	0	0	0
x_2	1	0	0	0	0
x_3	1	1	1	1	1



	z_0	z_1	z_2
y_0	1	0	0
y_1	1	0	0
y_2	1	0	0
y_3	1	0	0
y_4	1	1	1

(a) (b) (c) (d)



	z_0	z_1	z_2
x_0	1	0	0
x_1	1	0	0
x_2	1	0	0
x_3	1	1	1

(e) (f)

Figure 7: Example for the propagation of definite relationships.

- $x_1 \sqsubseteq y_0 \;\wedge\; y_0 \sqsubseteq \{z_0\} \rightarrow x_1 \cap z_1 = \phi \wedge x_1 \cap z_2 = \phi$
- $x_2 \sqsubseteq y_0 \;\wedge\; y_0 \sqsubseteq \{z_0\} \rightarrow x_2 \cap z_1 = \phi \wedge x_2 \cap z_2 = \phi$
- $x_3 \sqsubseteq \{y_0, y_1, y_2, y_3, y_4\} \;\wedge\; \{y_0, y_1, y_2, y_3, y_4\} \sqsubseteq \{z_0, z_1, z_2\}$

  x_3 has no empty intersections with components in Z.

Refining the above constraints, we get the following intersection matrix.

	z_0	z_1	z_2
x_0	1	-	a_1
x_1	1	0	0
x_2	1	0	0
x_3	1	1	a_2

Comparing the resulting matrix above with the matrices in figure 5, it can be seen that the result matrix corresponds to two possible relationships between objects x and z, namely the relationships R_3 and R_5.

A different conclusion is obtained if the relationship between objects y and z is as shown in figure 7(a). The composition of the relationships between x, y and z in this case will result in the definite matrix in figure 7(b) which corresponds to R_5 in figure 5.

3 Representation and Reasoning over Orientation Relations

Orientation spaces are defined using a similar strategy as used for topological spaces above. Adjacency between objects and the semi-infinite orientation areas are explicitly represented for each object. Orientation relationships between two objects are defined by the intersection between components in the object spaces.

Several schemes exist for the division of space to represent areas of acceptance for each orientation such as conical or rectangular. The approach defined here is independent of the scheme used to divide the space. Space divisions used in this paper are chosen for clarity and readability. More complex divisions can be treated in a similar way.

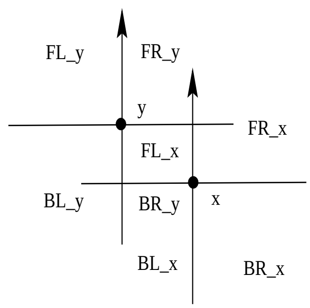
For simplicity object shapes are approximated by points in the following examples. Consider the orientation relations in figure 8(a) for an intrinsic frame of reference. The components of spaces X and Y are as follows: $X = x \cup FR_x \cup FL_x \cup BR_x \cup BL_x, Y = y \cup FR_y \cup FL_y \cup BR_y \cup BL_y$ where FR_i, FL_i, BR_i, BL_i denote the orientation relations: *Front-Right*, *Front-Left*, *Back-Right* and *Back-Left* respectively. The intersection matrix corresponding to this relation is shown in figure 8(b).

Both the relationship and its converse are needed to completely define the orientation relation in the case of the intrinsic frame of reference. For example, in figure 8(a), the relationship between objects x and y is defined by $BR(x, y) \wedge FL(y, x)$. If either of the objects rotates around itself, its relative relationship with the other object shall change as well, as shown in figure 9. In 9, object x has changed its orientation and hence also has changed its relationship with y to be: $BR(x, y) \wedge BL(y, x)$.

3.1 Examples of Qualitative Reasoning with Orientation Relations

For the sake of simplicity, the objects and the bounding lines of the orientations areas are omitted. This does not affect the reasoning process in the examples given or the features of the formalism since both general constraints are preserved for the semi-infinite areas. A mapping between non-empty intersections of space and the corresponding possible relations is given in table 10.

Each cell in the table contain the relations where there is a non-empty intersection between the corresponding components. If a relation is missing in a cell, then the intersection between the corresponding components is empty for the missing relation. For example, the highlighted cell in the table correspond-

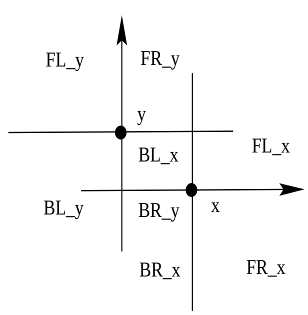


(a)

	x	FR_x	FL_x	BR_x	BL_x
y	0	0	1	0	0
FR_y	0	1	1	0	0
FL_y	0	0	1	0	0
BR_y	1	1	1	1	1
BL_y	0	0	1	0	1

(b)

Figure 8: (a) Example of an intrinsic orientation relation and its corresponding intersection matrix in (b). The arrow on the figure denotes the front of the object.



(a)

	x	FR_x	FL_x	BR_x	BL_x
y	0	0	0	0	1
FR_y	0	0	1	0	1
FL_y	0	0	0	0	1
BR_y	1	1	1	1	1
BL_y	0	0	0	1	1

(b)

Figure 9: (a) Changing the body orientation of object x gives a different relationship defined by the matrix in (b).

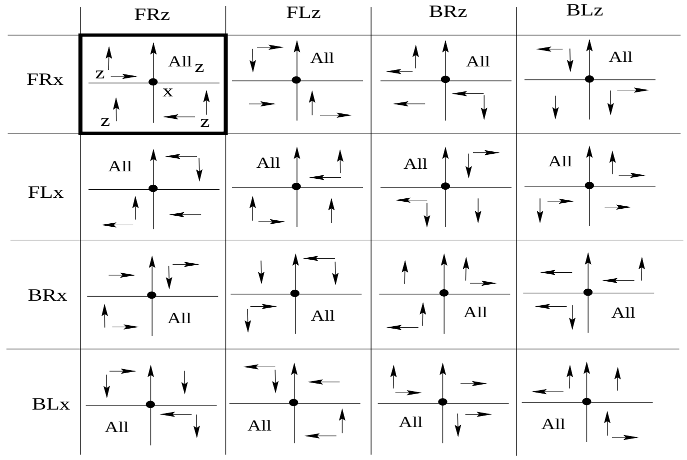


Figure 10: Correspondence between the intersection of the components and the relations in the intrinsic frame of reference. The highlighted cell entry is explained in the text. The cross represents the space of x, and the small arrows represent the front direction of object z.

ing to the components FR_x and FR_z is interpreted as follows: if we know that the intersection of the components FR_x and FR_z is not empty, then the relation between objects x and z could be either of the following:

a. $FR(z,x) \wedge (BL(x,z) \vee BR(x,z) \vee FL(x,z) \vee FR(x,z))$. Another way of expressing this is: $FR(z,x) \wedge All(x,z)$, or,

b. $BR(z,x) \wedge (FL(x,z) \vee FR(x,z))$, or,

c. $BL(z,x) \wedge FR(x,z)$, or,

d. $FL(z,x) \wedge (BR(x,z) \vee FR(x,z))$

Example: Propagation of Definite Compositions

Consider the simple example of composing the relationships: $FL(y,x) \wedge FL(x,y) \wedge BR(y,z) \wedge BR(z,y)$. The relationships and their corresponding intersection matrices are shown in figure 11(a) and (b).

The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- FR_y intersections:

$$\{FL_x, BL_x\} \sqsupseteq FR_y \sqsubseteq \{BR_z, BL_z\} \quad \rightarrow \quad (FL_x \cap BR_z \neq \phi \vee FL_x \cap BL_z \neq \phi)$$
$$\wedge \quad (BL_x \cap BR_x \neq \phi \vee BL_x \cap BL_z \neq \phi)$$

- FL_y intersections:

$$\{X\} \sqsupseteq FL_y \sqsubseteq \{BR_z\} \quad \rightarrow \quad (BR_z \cap \{X\} \neq \phi)$$

Note that the result of this composition can only identify the relative position of x to z ($BR(x,z)$), but not vice versa.

- BR_y intersections:

$$\{FL_x\} \sqsupseteq BR_y \sqsubseteq \{Z\} \quad \rightarrow \quad (FL_x \cap \{Z\} \neq \phi)$$

From this constraint it can be deduced that the relation between z and x is $FL(z,x)$.

- BL_y intersections:

$$\begin{aligned} \{FR_x, FL_x\} \sqsupseteq BL_y \sqsubseteq \{FR_z, BR_z\} \quad &\rightarrow \quad (FR_x \cap FR_z \neq \phi \vee FR_x \cap BR_z \neq \phi) \\ &\wedge \quad (FL_x \cap FR_z \neq \phi \vee FL_x \cap BR_z \neq \phi) \end{aligned}$$

Note the intersections of the components FL_y and BR_y have fully identified the composed relation, namely, $BR(x,z) \wedge FL(z,x)$. In this case, we don't need to apply rule 2. However for completeness the propagation of constraints by rule 2 are as follows:

- $\{FR_x\} \sqsubseteq (FL_y \cup BL_y) \sqsubseteq \{BR_z, FR_z\} \rightarrow FR_x \cap \{FL_z, BL_z\} = \phi$
- $\{BR_x\} \sqsubseteq FL_y \sqsubseteq \{BR_z\} \rightarrow BR_x \cap \{FR_z, FL_z, BL_z\} = \phi$
- $\{BL_x\} \sqsubseteq \{FL_y, FR_y\} \sqsubseteq \{BR_z, BL_z\} \rightarrow BL_x \cap \{FR_z, FL_z\} = \phi$
- FL_x has no empty intersections since $l' = l$.

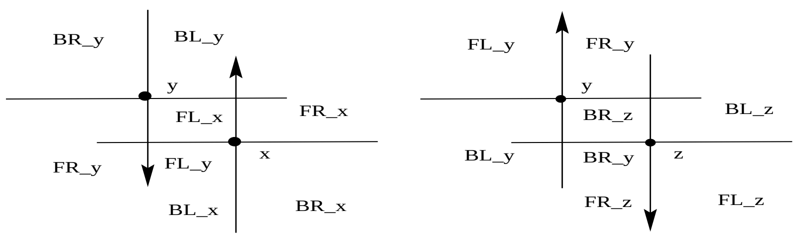
Grouping the above constraints, we get the intersection matrix in figure 11(c). Using table 10, it can be seen that the result matrix corresponds to the relationships $BR(x,z) \wedge FL(z,x)$ as in figure 11(d).

Example: Propagation of Indefinite Compositions

Consider the relationships in figure 12: $FL(y,x) \wedge FL(x,y) \wedge FR(z,y) \wedge FR(y,z)$. The corresponding intersection matrices are shown in (b). The reasoning rules are used to propagate the intersections between the components of objects x and z as follows. From rule 1 we have,

- FR_y intersections:

$$\{Z\} \sqsupseteq FR_y \sqsubseteq \{FL_x, BL_x\} \quad \rightarrow \quad (FR_z \cap FL_x \neq \phi \vee FR_z \cap BL_x \neq \phi) \qquad (a1)$$



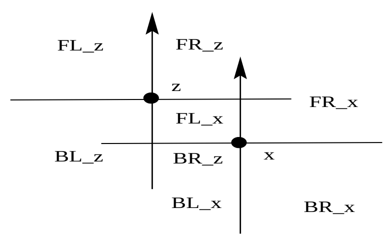
(a)

FR_x	FL_x	BR_x	BL_x		FR_z	FL_z	BR_z	BL_z
0	1	0	1	FR_y	0	0	1	1
1	1	1	1	FL_y	0	0	1	0
0	1	0	0	BR_y	1	1	1	1
1	1	0	0	BL_y	1	0	1	0

(b)

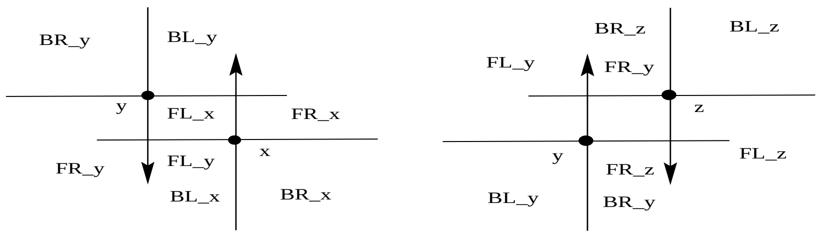
	FR_x	FL_x	BR_x	BL_x
FR_z	1	1	0	0
FL_z	0	1	0	0
BR_z	1	1	1	1
BL_z	0	1	0	1

(c)



(d)

Figure 11: (a) Composing the relationships $FL(x,y) \wedge FL(y,x)$ and $BR(y,z) \wedge BR(z,y)$. (c) Corresponding intersection matrices. (d) Resulting propagated constraints. (e) Corresponding (definite) relationship.



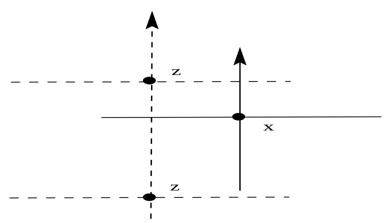
(a)

FR_x	FL_x	BR_x	BL_x		FR_z	FL_z	BR_z	BL_z
0	1	0	1	FR_y	1	1	1	1
1	1	1	1	FL_y	1	0	1	0
0	1	0	0	BR_y	1	1	0	0
1	1	0	0	BL_y	1	0	0	0

(b)

	FR_z	FL_z	BR_z	BL_z
FR_x	1, b_1	0	b_1	0
FL_x	a_1, b_2, 1	a_2, 1	a_3,b_2	a_4
BR_x	b_3	0	b_3	0
BL_x	a_1, b_4	a_2	b_4, a_3	a_4

(c)



(d)

Figure 12: (a) Composing the relationships $FL(x,y) \wedge FL(y,x)$ and $FR(y,z) \wedge FR(z,y)$. (b) Corresponding intersection matrices. (c) Resulting propagated constraints. (d) Corresponding (indefinite) relationships.

$$\wedge \quad (FL_z \cap FL_x \neq \phi \vee FL_z \cap BL_x \neq \phi) \qquad (a2)$$

$$\wedge \quad (BR_z \cap FL_x \neq \phi \vee BR_z \cap BL_x \neq \phi) \qquad (a3)$$

$$\wedge \quad (BL_z \cap FL_x \neq \phi \vee BL_z \cap BL_x \neq \phi) \qquad (a4)$$

- FL_y intersections:

$$\{FR_z, BR_z\} \sqsupseteq FL_y \sqsubseteq \{X\} \quad \rightarrow \quad (FR_x \cap FR_z \neq \phi \vee FR_x \cap BR_z \neq \phi) \qquad (b1)$$

$$\wedge \quad (FL_x \cap FR_z \neq \phi \vee FL_x \cap BR_z \neq \phi) \qquad (b2)$$

$$\wedge \quad (BR_x \cap FR_z \neq \phi \vee BR_x \cap BR_z \neq \phi) \qquad (b2)$$

$$\wedge \quad (BL_x \cap FR_z \neq \phi \vee BL_x \cap BR_z \neq \phi) \qquad (b2)$$

- BR_y intersections:

$$\{FR_z, FL_z\} \sqsupseteq BR_y \sqsubseteq \{FL_x\} \quad \rightarrow \quad (FL_x \cap FR_z \neq \phi \wedge FL_x \cap FL_z \neq \phi)$$

- BL_y intersections:

$$\{FR_z\} \sqsupseteq BL_y \sqsubseteq \{FR_x, FL_x\} \quad \rightarrow \quad (FR_z \cap FR_x \neq \phi \wedge FR_z \cap FL_x \neq \phi)$$

Applying rule 2 we get the following,

- $FL_z \sqsubseteq \{FR_y \cup BR_y\} \sqsubseteq \{FL_x, BL_x\} \rightarrow FL_z \cap FR_z = \phi \wedge FL_z FL_z \cap BR_x = \phi$
- $BL_z \sqsubseteq FR_y \sqsubseteq \{FL_x, BL_x\} \rightarrow BL_z \cap FR_x = \phi \wedge BL_z \cap BR_x = \phi$

Refining the above constraints, we get the intersection matrix in figure 12(c). Using the table 10, we get the possible relations in figure 12(d). Note that the conditions: $(a_1), (a_2), (b_1)$ and (b_2) are satisfied by definite intersections. The process of mapping the propagated intersections into possible relations in the table is carried out by finding the intersection of the set of relations corresponding to cells of value 1 in the matrix with the complement of the set of relations corresponding to cells of value 0 in the matrix. This process is demonstrated in figure 13. In 13(a) the intersection of the set of the relations corresponding to cells of value 1 is shown and in (b) the result from (a) is intersected with the complements of the sets of relations corresponding to cells of value 0.

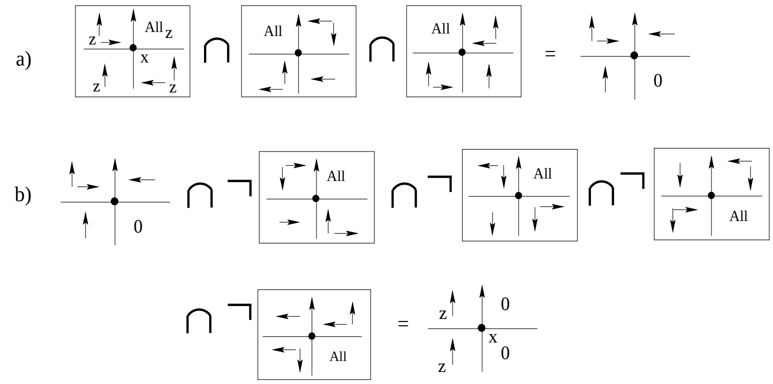


Figure 13: The process of mapping the constraints propagated by the reasoning rules to the set of possible relations, namely, $(FR(x,z) \wedge BL(z,x)) \vee (BR(x,z) \wedge FL(z,x))$. The figure is explained in the text.

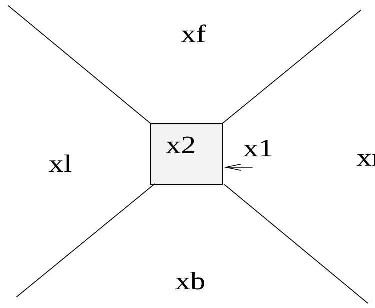
The result of the composition is indefinite and consequently the relative positions of the objects is ambiguous. The possible resulting relations between objects x and z are: $(FR(x,z) \wedge BL(z,x)) \vee (BR(x,z) \wedge FL(z,x))$

4 Possible Extensions of the Approach

In this section, the formalism proposed in extended in two ways. Firstly, the reasoning process is enhanced by considering multiple types of qualitative positional information at once, namely, combining topological and orientation information. Secondly, the application of the approach in assembly problems is studied, by enhancing the method with semi-quantitative shape information.

4.1 Combined Reasoning with Topological and Orientation Spaces

As can be expected, reasoning with more than one type of qualitative relationships would produce more precise results. One way of handling multiple types of relations using the representation and reasoning approaches above is by overlaying both the orientation and topological spaces for the objects. Hence, the combined space would contain both the object components as well as the orientation areas. Orientation areas around the object could either be defined using a representative point on the object (e.g. its centre)[], or using the minimum bounding rectangle of the object. The example in figure 14 illustrates the space components in the later case. Hence, spatial reasoning is carried out in similar fashion as above. Figure 15 shows the composition of the following relation-



x_1					
1	x_2				
1	0	x_f			
1	0	1	x_l		
1	0	0	1	x_b	
1	0	1	0	1	x_r

Figure 14: Representation of the combined topological and orientation spaces.

ships between objects x, y and z: $touch(x, y)$ *and* $front(y, x) \wedge disjoint(y, z)$ *and* $back(y, z)$. The composition would yield the definite relations:$disjoint(x, z)$ *and* $back(x, z)$. Note that the composition would have been indefinite if only the topological relationships were used; $touch(x, y) \wedge disjoint(y, z) \rightarrow disjoint(x, z) \vee touch(x, z) \vee overlap(x, z) \vee inside(x, z) \vee inside - and - touch - boundary(x, z)$.

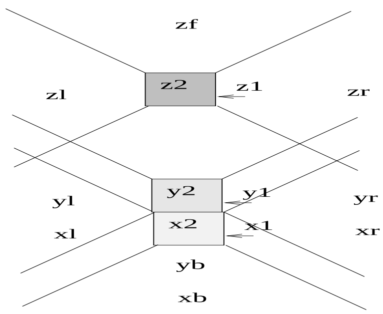
Also, proximity and relative size of the objects are important factors, which must be considered especially when the objects are very close or are in containment relation [].

4.2 Topological Structuring using Qualitative Height Information

Given a network of objects and relations, reasoning tasks can be one or more of the following:

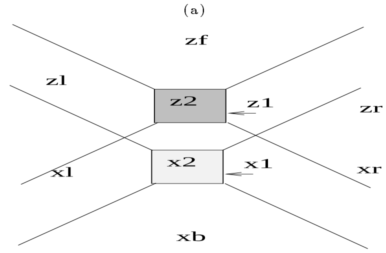
- Finding all feasible relations (minimal labelling problem).
- Finding a consistent scenario.
- Finding all consistent scenarios.
- Other reasoning tasks such as determining whether given spatial specifications are consistent.

In this section, qualitative topological reasoning is used for finding all consistent scenarios in the context of simple assembly problems. In CSG, any solid object could generally be constructed from an initial basic shape by adding or taking away other shapes in a sequence of operations and transformations. Here, this idea is borrowed where an object is represented qualitatively by a basic shape into which a set of holes are drilled and onto which a set of protrusions are added as shown in figure 16(a). A semi-qualitative representation is used by taking surface heights into account, where holes are represented by negative



z_r	z_b	z_l	z_f	z_2	z_1		x_1	x_2	x_f	x_l	x_b	x_r
0	1	0	0	0	0	y_1	1	0	1	0	0	0
0	1	0	0	0	0	y_2	0	0	1	0	0	0
1	0	1	1	1	1	y_f	0	0	1	0	0	0
0	1	1	0	0	0	y_l	0	0	1	1	0	0
0	1	0	0	0	0	y_b	1	1	0	1	1	1
1	1	0	0	0	0	y_r	0	0	1	0	0	1

(a)



	x_1	x_2	x_f	x_l	x_b	x_r
z_1	0	0	1	0	0	0
z_2	0	0	1	0	0	0
z_f	0	0	1	0	0	0
z_l	0	0	1,b	b	0	0
z_b	1	1	d, 1, a	1,a	1	1,d
z_r	0	0	1, c	0	0	c

(b)

Figure 15: (a) Composing $touch(x, y)$ *and* $front(y, x) \wedge disjoint(y, z)$ *and* $back(y, z)$. (b) Definite result: $disjoint(x, z)$ *and* $back(x, z)$

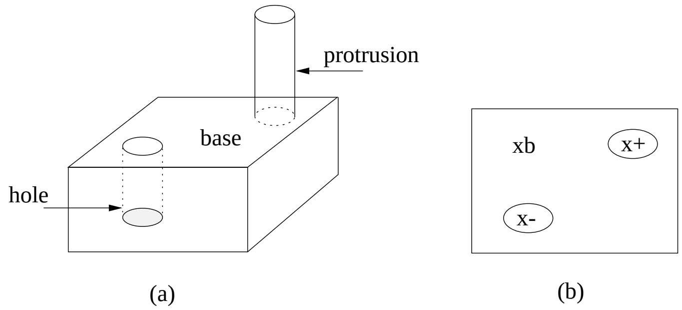


Figure 16: Qualitative representation using surface heights.

regions and protrusions by positive ones, as shown in 16(b). The basic shape is denoted base region (or zero-height region). Those regions can be established by constructing orthographic views of the object or by slicing the object at the level of its base region component. To determine all possible assembly scenarios, two steps need to be applied.

a. Topological representation of objects using surface heights as above. This step includes the definition of all relationship constraints between object components and forming a constraint network.

b. Topological reasoning over the network, applied as a constraint satisfaction problem to find all consistent scenarios.

4.2.1 Topological representation using surface heights

For an object x, let the component x_b denote its base region, ${x_i}^-$ denote a hole region, and ${x_j}^+$ denote a protrusion region. Object x can then be defined as: $x = x_b \bigcup_{i=0}^{n} {x_i}^- \bigcup_{j=0}^{m} {x_j}^+$

The following constraints apply between the components of the object. All possible relationships between simple regions are shown in figure 17.

a. Between x_b and ${x_i}^-$: $R(x_b, {x_i}^-) \in \{contain, b - contain\}$

b. Between x_b and ${x_j}^+$: $R(x_b, {x_j}^+) \in \{contain, b - contain, overlap, touch\}$

c. Between ${x_i}^-$ and ${x_k}^-$: $R({x_i}^-, {x_k}^-) \in \{disjoint\}$ and $0 \leq k \leq n$ and $k \neq i$.

d. Between ${x_j}^+$ and ${x_l}^+$: $R({x_j}^+, {x_l}^+) \in \{disjoint\}$ and $0 \leq l \leq m$ and $l \neq j$.

e. Between ${x_i}^-$ and ${x_j}^+$: $R({x_i}^-, {x_j}^+) \in \{disjoint, touch, contain\}$

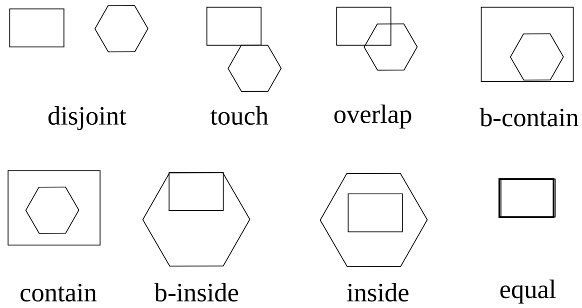


Figure 17: The set of all possible relationships between two simple regions.

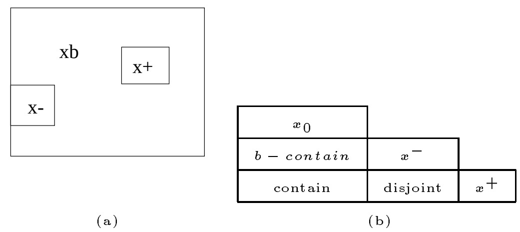


Figure 18: Topological representation using constraint networks.

Applying the above constraints, the topology of surfaces can represented as in the example shown in figure 18. Note that the relations are aread as cell(column, row).

4.2.2 Qualitative reasoning in Assembly Problems

The basic constraint in assembling any two faces of objects is that of contact, i.e. the faces must touch. Using the above topological representation of faces, this constraint is interpreted to a set of constraint between different objects components. For two objects x and y the constrains are:

a. $R(x_b, y_b) \in \{touch, overlap, equal, inside, contain, b - contain, b - inside\}$

b. $R({x_i}^-, {x_i}^-, {y_p}^-) \in \{U\}$ where U is the set of all possible relations.

c. $R({x_j}^+, {y_p}^+) \in \{touch, disjoint\}$

d. $R({x_i}^-, {y_p}^+) \in \{disjoint, touch, equal, contain, b - contain\}$

e. $R(x_b, {y_p}^-) \in \{U\}$

f. $R(x_b, {y_p}^+) \in \{disjoint, touch, inside\}$ The *inside* relationships is only valid provided, $R({y_p}^+, {x_i}^-) \in \{inside, b - inside, equal\}$

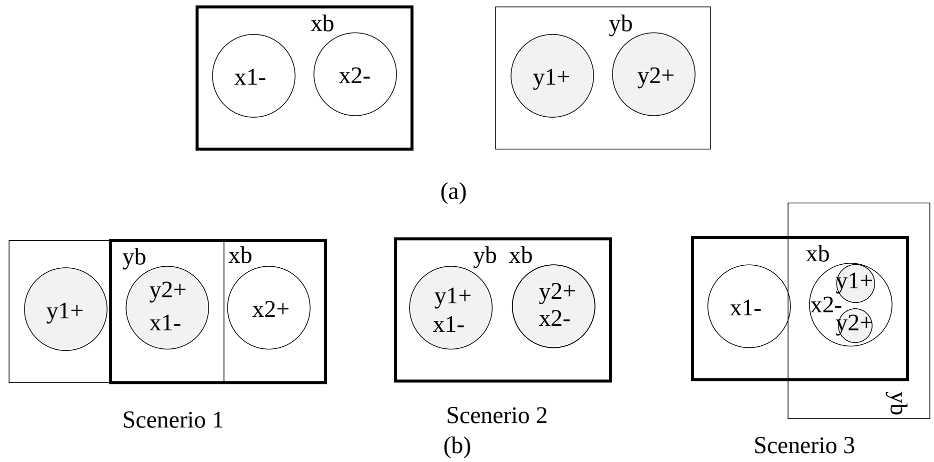


Figure 19: (a) Assembling two lego bricks x and y. (b) Three topologically possible scenarios.

g. $R(x_j{}^+, y_p{}^-) \in \{equal, inside, b-inside\}$ This is a global constraints which must be satisfied at least once.

Example:

Consider the simple example of a child assembling two pieces of Lego bricks. If two faces of the bricks are considered as shown in figure 19(a), one with two protrusions, and the other with two holes. Applying the above constraints will produce three different scenarios depicted in figure 19(b). Scenario 3 is not possible and can be excluded if size constraints are taken into account, $larger(y_1{}^+ + y_2{}^+), (x_1{}^- \vee x_2{}^-))$. Using different types of qualitative relations would need to be used if the minimum possible solution set is to be derived qualitatively.

5 Comparison with Related Work

Several approaches are reported in the literature for the representation and reasoning over topological and orientation relations. The main advantage of the representation method proposed is its uniformity. The same methodology is used for the definition of simple and complex objects and is applied consistently in the orientation and topological spaces. The method is also adaptable, where different levels of representation can be devised by hiding or revealing the details of objects as required. The method is therefore well adapted for use as a basis for a spatial reasoning formalism.

The representation of complex regions has been addressed in many works. In [CBGG97], Cohn et al extended the RCC formalism to handle concave regions, and regions with holes (doughnut shapes). New axioms and theories had to be devised to define the new shapes. The main drawback of this approach is its complexity, as new, possibly considerable, extensions of the formalism have to be devised with every new shape considered.

In [EH90, EF91], Egenhofer et al used point-set topology to define simple regions, using three components, boundary, interior and exterior. The method proposed here deviates from their work in one important respect which has far-reaching implications. We relaxed the constraint on the object components to be any possible set of components which satisfies the main assumptions behind the formalism. The notions of boundaries, interiors and exteriors were dropped and the notion of object and space components is used instead. Egenhofer's method is limiting and could not be extended to handle complex objects. Other methods were devised in [ECP94] to define regions with holes, through the definition of spatial relationships between simple regions and no extension for the method was proposed for the definition of irregular, or concave regions.

The work of Clementini and De Felice [CDF94] follows closely the method of Egenhofer, and provides a definition for regions with holes using boundaries, interiors and exteriors. His method carries the same limitations as those of [EH90]. In another work [CDFC95], Clementini et al addressed the issue of defining composite regions for use in spatial query languages, by defining explicit relationships between all the components in the object, in the same way, regions with holes were defined in [ECP94].

Coenen and Pepijin [CP98] proposed an ontology for objects and relationships in spatio-temporal domains. They assumed the space to be consisting of sets of points and used set-theoretic notions to define objects in that space. Their approach is distinctive from the above where space is considered to be discrete, not continuous. The method was used to define a general "object" and quantitative identifiers are used to qualify the object properties.

Our approach is a an example of the constraint-driven approach where a spatial relationship is defined by a set of intersection constraints between the object components. As mentioned earlier, the approach is a fundamental variation from that proposed by Egenhofer and Franzosa [Ege89].

The approach developed by Randell et. al [RCC92a] is an example of the relation-driven approach to representing topological relationships, where a set of axioms for defining every needed relationship have to be devised. For example, the definition of the `overlap` relationship between two simple regions is :

$Overlap(x,y) \leftarrow Part(z,x) \wedge Part(z,y)$ and a set of constraints are used for defining the relationship $Part(x,y)$.

Approaches for representing orientation relationships can be classified under two main categories: *projection-based* approaches and *space-division* approaches.

- In the projection-based approaches, objects are projected on the x and y axes, dividing the axes up into several ordered parts. By comparing the order of these projected parts, the orientation relation is inferred. Reasoning in these approaches exploits the interval algebra (Allen's transitivity tables) from the temporal domain [Gue89, CJ86, Fra92]

- The space-division approaches are based on dividing the space around the object into semi-infinite acceptance areas. Reasoning in these approaches utilises spatial composition tables which are usually built manually, by a visual process, for every object type or space granularity considered.

Fewer works exist for defining and reasoning over other types of relationship. In [HCDF95] proximity is defined by distance values and reasoning is carried out by vector sums with respect to specific orientation between objects. In Gahegan [Gah95] a fuzzy set membership relations is used to reason about degrees of closeness.

In general, approaches to handling orientation relations are limited in their expressiveness, due to their inability to represent different types of orientation relations for different object dimensions without restricting the space resolution or division. Also, building composition tables manually affects the tractability of the reasoning process in the case of high space resolutions. In general, approaches are specific and none of the existing approaches offers a unified method for handling different types of spatial relations, which is the main aim of the work presented here.

Approaches to spatial reasoning in the literature can generally be classified into a) using *transitive propagation* and b) using *theorem proving*.

- Transitive propagation: In this approach the transitive property of some spatial relations is utilised to carry out the required reasoning. This applies to the *order relations*, such as `before`, `after` and $(<,=,>)$ (for example, $a < b \ \wedge \ b < c \rightarrow a < c$), and to the subset relations such as `contain` and `inside` (for example, $inside(A,B) \wedge inside(B,C) \rightarrow inside(A,C)$, $east(A,B) \wedge east(B,C) \rightarrow east(A,C)$).

Transitive property of the subset relations was employed by Egenhofer [Ege94] for reasoning over topological relationships. Transitive property of the order relations has been utilised by Mukerjee & Joe [MJ90], Guesgen [Gue89], Chang & Lu [CL84], Lee & Hsu [LH91] and Papadias & Sellis [PS92].

Although order relations can be utilised in reasoning over point-shaped objects, they cannot be directly applied when the actual shapes and proximity of objects are considered.

- Theorem proving (elimination): Here, reasoning is carried out by checking every relation in the full set of *sound* relations in the domain to see whether it is a valid consequence of the composition considered (theorems to be proved) and eliminating the ones which are not consistent with the composition [CRCB93].

  Bennett [Ben94] have proposed a propositional calculus for the derivation of the composition of topological relations between simple regions using this method. However, checking each relation in the composition table to prove or eliminate is not possible in general cases and is considered a challenge for theorem provers [RCC92b].

In general the limitation of all the methods in the above two approaches are as follows:

- Spatial reasoning is studied only between objects of similar types, e.g. between two lines or two simple areas. Spatial relations exist between objects of any type and it is limiting to consider the composition of only specific object shapes.
- Spatial reasoning was carried out only between objects with the same dimension as the space they are embedded in, e.g. between two lines in 1D, between two regions in 2D, etc.
- Spatial reasoning is studied mainly between simple object shapes or objects with controlled complexity, for example, regions with holes treated as concentric simple regions. None of the methods in the literature have been presented for spatial reasoning between objects with arbitrary complexity.

6 Conclusions

A general approach is presented for qualitative spatial representation and reasoning. No limitations on the complexity of the objects are imposed [AJ97]. It is based on a uniform representation of the topology of the space as a connected set of components. A structure called adjacency matrix is proposed to capture the topology of objects with different complexity. It is shown how topological spatial relations can be uniquely defined. The reasoning method consists of a set of two general constraints to govern the spatial relationships between objects in space, and two general rules to propagate relationships between objects in space. The reasoning process is general and can be applied on any types of objects with random complexity. It is also simple and is based on the application of two rules for the propagation of empty and non-empty intersections between object components.

The method has been applied to orientation spaces and extended to cater for hybrid reasoning where topological and orientation spaces are used concurrently. A possible extension of the method was also presented which takes into account qualitative surface height information.

It can be concluded that significant benefits are envisaged if spatial reasoning systems are enhanced by qualitative manipulation of different types of positional information.

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