

# The Multi-Objective Uncapacitated Facility Location Problem for Green Logistics

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**Abstract**—Traditionally, the uncapacitated facility location problem (UFLP) is solved as a single-objective optimization exercise, and focusses on minimizing the cost of operating a distribution network. This paper presents an exploratory study in which the environmental impact is modelled as a separate objective to the economic cost. We assume that the environmental cost of transport is large in comparison to the impact involved in operating distribution centres or warehouses (in terms of CO<sub>2</sub> emissions, for example). We further conjecture that the full impact on the environment is not fully reflected in the costs incurred by logistics operators. Based on these ideas, we investigate a number of “what if?” scenarios, using a Fast Nondominated Sorting Genetic Algorithm (NSGA-II), to provide sets of nondominated solutions to some test instances. The analysis is conducted on both two-objective (economic cost versus environmental impact) and three objective (economic cost, environmental impact and uncovered demand) models. Initial results are promising, indicating that this approach could indeed be used to provide informed choices to a human decision maker.

## I. INTRODUCTION

In logistics network design, the facility location problem (FLP) involves strategic level decisions with planning horizons of several years. It identifies the optimum number and locations of depots or warehouses, in a distribution network, in which deliveries are made to local customers and/or goods are collected from local suppliers (see Figure 1). In addition, each customer (or supplier) must be assigned to exactly one depot. In practice there are many variations of the FLP, for example, storing inventory before it is transported to customers, or including transshipment points, where the goods are reloaded from the supplier to be forwarded to the retail stores. Also the FLP is easily adapted to identify the optimum number of recycling or collection facilities in a network. The facility location problem is not new to academia and has a very rich literature. In [7] the role of facility location models within a supply chain context was described as an “extremely interesting and fruitful application area domain”.

Typical formulations for the FLP aim to minimize cost as a single objective. In this approach, the total cost is frequently expressed as a sum of various component expenses, most simply as transportation and fixed costs. However, in many practical situations, the optimum design may involve dealing

with multiple and sometimes conflicting objectives. In a recent survey [7], 120 articles published in the last decade were categorized: 75% had a single cost minimization objective, 16% had a single profit maximization objective and only 9% were modelled with multiple and conflicting objectives. The multiple objectives mentioned include resource utilization and customer responsiveness, in addition to the standard economic objectives. Recent concerns regarding climate change, however, have shifted the focus of modelling to incorporate environmental objectives.

There are different techniques for solving problems involving multiple objectives. Classical multi-objective optimization methods, such as  $\epsilon$  - constrained [14], and goal programming [15] suggest a way of transforming the multiple objectives into a single one before solving the problem using a single objective optimization algorithm [13]. The usefulness of the single solution obtained following the transformation, however, depends on making suitable choices of the parameters in the conversion model. We can see that some recent multi-objective models which incorporate environmental measures are solved using classical methods. For example, [8] present a generic mathematical programming model for assisting the strategic long range planning and design of a bulk chemical network. Their multi-objective mixed-integer programming problem is solved using the  $\epsilon$  -constraint method [14], in which they minimize the environmental impact resulting from the operations of the entire network, and simultaneously maximize the profitability of the network. Another example, assessing the trade-offs between cost and environmental impact, is described in [12], where the re-organization of a European pulp and paper logistic network is described. In their model, the locations of the facilities, such as paper recycling and paper production, are fixed, which leads them to solve only the allocation problem, unlike the FLP we solve in the present paper. They use multi-objective optimization to make their assignments.

From our recent review [11], like [7], we identified only a very small number of multi-objective models with environmental objectives, and these were solved predominantly using classical multi-objective methods. Classical techniques rely on *a priori* judgements regarding the relative importance of the various component objectives. In contrast, there are

other approaches that do not rely on such assumptions and treat all objectives equally. Such techniques will generate a set of solutions, with the objectives traded off in different ways, instead of a single optimum with respect to a pre-defined (perhaps arbitrary) trade-off situation. In this way it is possible to provide a decision maker with sufficient choices to make an informed judgement when trading off the relative merits of the conflicting objectives. In this research we explore an elitist multi-objective evolutionary algorithm for the strategic modelling of a logistics network, where economic and environmental objectives are considered simultaneously.

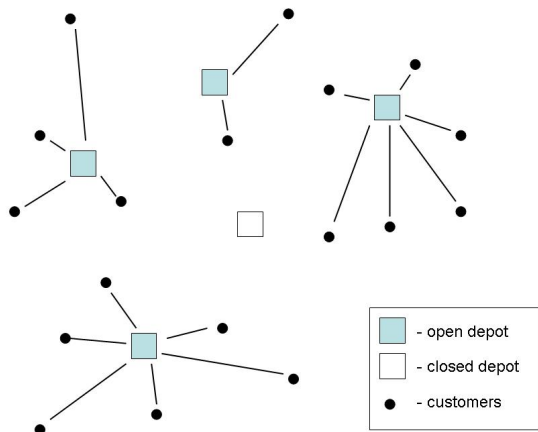


Fig. 1. The Uncapacitated Facility Location Problem

In our paper we apply the Nondominated Sorting Genetic Algorithm (NSGA-II) [3] to a multi-objective uncapacitated facility location problem (MOUFLP) in the context of green logistic design. Green logistics implies “an environmentally friendly and efficient transport distribution system” [10]. The uncapacitated facility location problem (UFLP) is the simplest form of FLP, and involves identifying which depots to open, assigning the customers to open depots, and has no constraints regarding the capacity of the facilities (Figure 1). Our multi-objective model has two different settings: two-objectives (min cost - min environmental impact) and three-objectives (min cost, min environmental impact and min uncovered demand). For this simple model, our environmental objective is formulated in a similar way to our objective measuring economic cost, and is made up of two components: depot costs and transportation costs. However, we weight these components differently for assessing the environmental impact, working under the assumption that the environmental cost of transport is large in comparison to the impact involved in operating distribution centres or warehouses (in terms of CO<sub>2</sub> emissions, for example). We further conjecture that the full impact on the environment is not reflected in the costs incurred by logistics operators. Based on these ideas, we investigate a number of “what if?” scenarios, by varying the relative weighting of the impact of transport versus depots on the environment to provide sets

of nondominated solutions to some test instances. This is an exploratory study aimed at investigating the potential of multi-objective optimization techniques for the FLP.

The remainder of the paper is organized as follows. Section II introduces our multi-objective model for the UFLP with an environmental objective and also describes the test data which we use in the study. Section III outlines the NSGA-II algorithm, and describes the operators and solution representation that we use. In Section IV we describe our experimental method, and in Section V we present our results. Finally, we summarize our main findings and suggest future work in Section VI.

## II. OUR MULTI-OBJECTIVE OPTIMIZATION MODEL

The main drivers in traditional logistics network design are to reduce total costs and improve customer service levels. Due to recent concerns regarding climate change, minimizing the environmental impact from depots and transport needs to be addressed as well. Our proposed multi-objective uncapacitated facility location problem incorporates those three goals. Therefore, the problem definition in this paper is a mixture of three mathematical programming formulations: the uncapacitated facility location problem, a revised UFLP with environmental weightings, and the maximal covering location problem (MCLP). The MCLP involves first asserting a global (ideal) maximum distance between customer and serving depot. Once customers have been assigned to depots, covered demand can be measured as the percentage of customer demand met within the given distance radius. Customers assigned to depots that are further away than this maximum, represent uncovered demand (100 % - covered-demand-percentage).

The UFLP and MCLP models we use have been adapted from [2] who originally based their formulation on [4]. [2] present a bi-objective UFLP (min cost - max coverage) in their paper. To solve the problem, they designed and implemented three different algorithms to obtain a good approximation of the Pareto frontier. The algorithms were based on the Nondominated Sorting Genetic Algorithm, the Pareto Archive Evolution Strategy and on mathematical programming.

### A. Problem formulation and objective functions

We will assume that the customers each have a certain demand and that transportation costs and fixed costs for the open depots are linear and additive.

We further assume that at least one depot from a set of depots will be open, and that each depot will serve its customers directly. The problem is to determine how many depots to locate, where to locate them and which depot serves which customer, in order to satisfy the three objectives: minimize cost, minimize environmental impact and minimize uncovered demand. Solving this problem requires two main routines: one to determine which depots to open, and the other to assign the customers to the open depots (the assignment rule), where each customer is assigned to exactly

one depot. The values of the two or three objectives can be computed once a network configuration has been defined.

The following notation is used in the formulation of the model:

$\tau = \{1\dots i\}$	set of potential depots;
$\gamma = \{1\dots j\}$	set of customers;
$c_{ij}$	transportation cost of attending demand from customer $j$ to depot $i$ ;
$f_i$	fixed cost for opening depot $i$ ;
$d_j$	demand of customer $j$ that could not be attended within $Dmax$ by particular depot $i$ ;
$h_{ij}$	the distance between depot $i$ and customer $j$ ;
$Dmax$	the maximal covering distance - the customers within this distance to an open depot are considered well served;
$\Theta$	set of depots that could not attend customer $i$ within the maximal covering distance $Dmax$ ;

The decision variables are:

$x_{ij}$	equals 1 if the whole demand of customer $j$ is attended by depot $i$ and 0 otherwise;
$y_i$	equals 1 if depot $i$ is chosen to operate and 0 otherwise;

The following objectives functions are considered simultaneously as part of the location design:

- *Minimising costs.* The objective is to find the best number and location of depots that minimizes total transportation and fixed costs. The first term represents the cost of attending demand of customers by the open depots and the second term represents the fixed facility cost of the open depots.

$$\text{minimize} \left[ \sum_{i \in \tau} \sum_{j \in \gamma} c_{ij} x_{ij} + \sum_{i \in \tau} f_i y_i \right] \quad (1)$$

- *Minimising uncovered demand.* The objective measures total uncovered demand as a sum of the demand of customers which could not be attended by depot within maximal covering distance.

$$\text{minimize} \left[ \sum_{j \in \gamma} d_j \sum_{i \in \Theta_j} x_{ij} \right] \quad (2)$$

- *Minimizing the environmental impact from transport and depots.* The objective is to find the best number and location of facilities that minimizes the total environmental impact from transportation and depots. This is essentially the same formulation as we use to minimize economic costs, but we introduce  $W_T$  and  $W_F$  to weight the transport and fixed costs, respectively, for environmental impact. In this model, higher values of  $W_T$  imply worse pollution from transport.

$$\text{minimize} \left[ \sum_{i \in \tau} \sum_{j \in \gamma} c_{ij} * W_T * x_{ij} + \sum_{i \in \tau} f_i * W_F * y_i \right] \quad (3)$$

where  $W_T$  is the factor which derives the environmental impact from transport in relation to transportation costs

and  $W_F$  is the factor which derives the environmental impact from depots in relation to fixed costs. For the present study we used following values:  $W_F = 1$  and  $W_T \in [1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24]$ .

- *Subject to following constraints:*

$$\sum_{i \in \tau} x_{ij} = 1, j \in \gamma \quad (4)$$

$$x_{ij} \leq y_i, j \in \gamma, i \in \tau \quad (5)$$

$$x_{ij} \in \{0, 1\}, j \in \gamma, i \in \tau \quad (6)$$

$$y_i \in \{0, 1\}, i \in \tau \quad (7)$$

Constraints 4 and 6 ensure that each customer is attended by only one depot. Constraint 5 assigns the customers to open depots. Constraints 6 and 7 define decision variables as binary.

For our analysis we looked at two different setting for objectives:

- 1) for the two-objective UFLP: minimizing costs and minimizing environmental impact;
- 2) for the three-objective UFLP: minimizing cost, minimizing environmental impact and minimizing uncovered demand.

## B. Test data.

From our research, we confirmed findings published in [2]: that there are no available MOUFLP test instances in the public domain for benchmarking. Especially for our research, we needed instances which consider environmental information. To begin our research, we obtained bi-objective problem instances (min cost and max coverage) from [2] of two different types: instance A and instance B. The difference between the instances is that the locations have been generated in different ways: uniformly distributed locations within a square (instance A) or depot locations chosen from customer locations (instance B). A and B instances come in three different sizes: 10 depot-25 customers, 30 depots-75 customers and 50 depots-150 customer. Each size also differs in its fixed depot cost structure (uniformly distributed (C1–C3) or the same fixed costs for all depots (C4–C6)). For example, instance A10-25C3 is an instance of type A with 10 available depots, 25 customers and a uniformly distributed fixed depot cost structure. In total, 26 different problem instances are provided. For our analysis, we have chosen one instance of each type, size and cost structure - in total 12 test instances. For example, the following instances of type A were used for analysis: A10–25C3, A10–25C6, A30–75C3, A30–75C6 etc.

The data sets described above model economic costs and coverage but not environmental costs. To include an environmental objective, we used the simple weighted model

described above, and applied it to the fixed costs and transportation costs of the “standard” UFLP. We know that the environmental impact from transport is closely related to fuel consumption. However, there are other factors that have an impact on the actual levels of emissions from transportation, such as the speed and acceleration of the vehicle; the load on the engine over the distance traveled; the type of fuel used, vehicle condition, engine size etc. [5]. Of course, all of these factors will also impact on economic cost. On the other hand, other expenses, such as vehicle maintenance, road tax, training costs, drivers’ wages etc., will not directly impact on the environment in the same way. Regarding depot costs, the environmental impact from depots comes from the electricity and gas consumption by the buildings. Economic costs for depots also include rent/rates and staffing costs. In this paper we will assume that transport has a relatively greater impact on the environment than depots, relative to economic costs, and we will use our simple model to explore various scenarios related to different weightings of the environmental impact of transport.

### III. THE EVOLUTIONARY MULTI-OBJECTIVE ALGORITHM: NSGA-II

The evolutionary Nondominated Sorting Genetic Algorithm NSGA-II [3] was chosen for implementation because it has all the qualities which are needed to be taken into consideration when solving a multi-objective problem. It is a well tested algorithm in academia. It is elitist (preserving the best solutions) and uses a diversification mechanism, called a *crowding distance*, to ensure the solutions are widely and evenly distributed. Fitness assignment in the algorithm uses ranking, based on fast non-dominated sorting (see Algorithm 2).

In the fast non-dominated sort procedure (see Algorithm 2), the algorithm uses the concept of domination (see Definition 1), where two chromosomes are compared on the basis whether one chromosome dominates another chromosome or not.

*Definition 1.* A solution  $x_1$  is said to dominate the other solution  $x_2$  ( $x_1 \prec x_2$ ), if both conditions 1 and 2 are true [13]:

- 1) The solution  $x_1$  is no worse than  $x_2$  in all objectives, or  $f_j(x_1) \not\prec f_j(x_2)$  for all  $j = \{1, 2, \dots, M\}$
- 2) The solution  $x_1$  is strictly better than  $x_2$  in at least one objective, or  $f_{\bar{j}}(x_1) \triangleleft f_{\bar{j}}(x_2)$  for at least one  $\bar{j} = \{1, 2, \dots, M\}$

In Algorithm 2, the crowding comparison operator ( $\leq_{nsga}$ ) compares two solutions and returns the fitter of the two as the “winner” (a binary tournament selection). It assumes that every solution  $i$  in the population has a non-dominant rank  $r_i$  and a local crowding distance  $cd_i$ .

*Definition 2.* The Crowded Tournament Selection Operator [13]: A solution  $i$  wins a tournament with another solution  $j$  if either of the two conditions below are true:

- 1) If solution  $i$  has a better rank, that is,  $r_i < r_j$

- 2) If they have the same rank but solution  $i$  has a better crowding distance than solution  $j$ , that is,  $r_i = r_j$  and  $cd_i > cd_j$

The NSGA-II algorithm for a multi-objective UFLP is adopted from [2] and [3] and operates as outlined in Algorithm 1. Firstly, an initial parent population  $P(0)$  of size  $N$  is created, at random. Each parent solution is encoded as a binary string (see below). For each chromosome in  $P(0)$ , the objectives (e.g. cost and impact) are evaluated by applying the assignment procedure (see below). Then, a fast non-dominated sort is applied to  $P(0)$  (see Algorithm 2), which assigns a “front number” to each solution which is equal to its non-dominant level, starting with 1 (1 is the best). Binary tournament selection in the parent population  $P(0)$  is followed by crossover and mutation to generate the child population  $C(0)$  of size  $N$ . Each child solution in  $C(0)$  is then evaluated.

Next, the following elitist procedure for  $t \geq 1$  described below is repeated for  $T$  generations. At the start of this, the parent and child populations are combined to form  $R(t) = P(t) \cup C(t)$  of size  $2 * N$  and a fast non-dominated sort is applied to  $R(t)$ . A new parent population,  $P(t + 1)$ , is then formed from  $R(t)$  by adding solutions beginning with the first front onward to make up a population of size  $N$ . Crowding distance is used to help make the last few selections, if addition of all individuals from a particular front would produce a population greater than  $N$ . Then, the child population  $C(t + 1)$  of size  $N$  is created from  $P(t + 1)$  by applying binary tournament selection, crossover and mutation. The overall complexity of the algorithm is  $O(mN^2)$ .

*Solution encoding.* Each solution for MOUFLP is encoded as a binary string of length equal to the total number of (potential) depots, where each bit indicates whether depot is open (value of 1) or closed (value of 0)(e.g. 1101100100). However, a solution also involves the assignment of customers to depots. This is performed by the assignment procedure described below.

*Assignment procedure.* In location models it is very important to decide how the customers are assigned to the particular facilities. In some circumstances, the assignment depends on the distance or travel time, in other cases it could depend on the range or quality of the products dispatched or collected. Our model incorporates a customer service level objective, therefore we used the assignment procedure described in [2], which tries to minimize cost without impacting on coverage. Provided a customer is located within a given maximum distance radius,  $Dmax$ , then that customer is assigned to the depot at the minimum transportation cost. If a customer cannot be covered (i.e., the nearest depot is further than  $Dmax$ ) then it is assigned to the depot with the smallest transportation cost, regardless of its distance. Ties are broken on a first-come-first-served basis.

*Mutation.* For each solution (chromosome), a random mutation pattern is generated. A uniform random number between 0 and 1 is generated for each position in the solution.

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**Algorithm 1** NSGA-II algorithm for MOUFLP

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**Begin:**

Randomly generate parent population  $P(0)$  of size  $N$   
Evaluate  $P(0)$  - calculate/record the value of objectives  
Fast non-dominated sort ( $P(0)$ )  
Generate child population  $C(0)$  of size  $N$  from  $P(0)$  by  
applying binary tournament selection with the selection  
criterion based on  $\leq_{nsga}$ , crossover and mutation  
Evaluate  $C(0)$  - calculate/record the value of objectives  
**while**  $t \leq T$  **do**  
     $R(t) = P(t) \cup C(t)$   
     $F$  = fast non-dominated sort  $R(t)$  (see Algorithm 2)  
    Crowding-distance assignment ( $F$ ) (see Algorithm 3)  
    Sort  $R(t)$  using  $\leq_{nsga}$  (see Definition 2)  
    Select  $P_{t+1}$  from sorted  $R_t [0 : N]$   
    Generate child  $C(t+1)$  of size  $N$  from  $P(t+1)$  by  
    applying binary tournament selection with the selection  
    criterion based on  $\leq_{nsga}$ , crossover and mutation  
    Evaluate  $C(t+1)$  - calculate/record the value of objec-  
    tives  
     $t = t + 1$   
**Return** all non-dominated solutions from first front  $F(1)$

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**Algorithm 2** Fast non-dominated sort ( $P$ )

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Input Parameters: population ( $P$ ), consisting of chromo-  
soms, e.g.  $p, q$

**Begin:**

**for each**  $p \in P$  **do**  
    **for each**  $q \in P$  **do**  
        **if** ( $p \prec q$ ) **then**  
             $S_p = S_p \cup \{q\}$  {if  $p$  dominates  $q$  - save it in set of  
            solutions  $S_p$ , which  $p$  dominates }  
        **else if** ( $q \prec p$ ) **then**  
             $n_p = n_p + 1$  {if  $q$  dominates  $p$  - keep the count of  
            the solutions dominating  $p$ }  
    **if**  $n_p = 0$  **then**  
         $F_1 = F_1 \cup \{p\}$  {if nobody dominates  $p$  then it joins  
        the first front }  
 $i = 1$   
**while**  $F_i \neq \emptyset$  **do**  
     $H = \emptyset$   
    **for each**  $p \in F_i$  **do**  
        **for each**  $q \in S_p$  **do**  
             $n_q = n_q - 1$  {reduce count of solutions which  
            dominates  $q$ }  
            **if**  $n_q = 0$  **then**  
                 $H = H \cup \{q\}$  {if  $n_q$  is zero then  $q$  joins list  $H$ }  
     $i = i + 1$   
     $F_i = H$  {form current front with members of  $H$  }  
**Return** a list of non-dominant fronts  $F$

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**Algorithm 3** Crowding-distance assignment ( $F$ )

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Input Parameters: solutions in front  $F$

**Begin:**

$l = |\tau|$  {number of solutions in front  $F$ }

**for each**  $i$  **do**

**set**  $F[i]_{distance} = 0$  {initialize distance for each solution}

**for each objective**  $m$  **do**

$F = \text{sort}(\tau, m)$  { sort using each objective value}

$F[1]_{distance} = F[l]_{distance} = \infty$  {assign large values to  
    the boundary solutions}

**for**  $i = 2$  **to**  $(l - 1)$  **do**

$F[i]_{distance} = F[i]_{distance} + (F[i+1].m - F[i-1].m)$

**Return** crowding distance for each point in front  $F$

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If this random number is less than mutation probability ( $p_m$ ), the gene is flipped either from 0 to 1 or 1 to 0. After initial experiments,  $p_m = 0.06$  was used for tuning mutation parameters in NSGA-II.

*Crossover.* Binary tournament selection was used to choose the parents for crossover, as described above. Two individuals are randomly selected from the parent population  $P(t)$  and the fitter one of the two is chosen as a parent, i.e., the one which wins the crowded tournament selection (see Definition 2). This means that the chromosome wins if it has higher non-domination level or if two chromosomes have the same non-domination level, then we choose the one that has a better crowding distance.

In some pilot experiments we compared three different crossover operators: one-point, two-point, uniform crossover, and also tried running NSGA-II with no crossover. The performances of the crossovers were then assessed using the  $S$  metrics described in [6]. For each crossover setting we ran 20 independent trials for two and three objectives on two test cases, A30-75C3 and A30-75C6. As a result of these experiments we chose two-point crossover for the two-objective problem and no crossover for the three objective problem, as in the latter case the crossover had no significant impact on the results.

#### IV. EXPERIMENTAL METHOD

The purpose of our experiments is to answer the following questions for each MOUFLP: the two-objective problem (min cost and min environmental impact) and the three-objective problem (min cost, min uncovered demand and min environmental impact):

- 1) Does this approach hold promise – do we obtain a reasonable trade-off front?
- 2) What happens to the solution set as we explore scenarios in which the environmental impact of transport increases disproportionately to its cost?
- 3) How do we select suitable trade-off solutions from the approximate Pareto front?

As previously mentioned, six instances of type A and six similar instances of type B were selected from the test data taken from [2]. Recall that these instances have

data for a two-objective problem (economic cost, coverage), and we applied our environmental weightings,  $W_T$  and  $W_F$ , to transport and fixed costs, respectively. The plan is to assess the environmental impact for a range of “what if?” scenarios, in which  $W_F = 1$  in all cases, and  $W_T \in [1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24]$ . We assume that the environmental impact of transport outweighs the environmental cost of maintaining depots. For each problem instance we performed 12 experiments, one for each of the above environmental factors for transport ( $W_T$ ). In each run, the initial solution was created randomly at the start.

The NSGA-II algorithm creates a child population from its parent population using fast non-dominated sort, crossover and mutation. The parent population ( $P_t$ ) and child population ( $C_t$ ) were set to the same size  $N=40$  for our experiments, and for each run, on every instance, the number of generations was 250.

The crossover probability ( $p_c$ ) was used to determine the number of chromosomes participating in the crossover. After initial experiments, we settled on  $p_c = 0.7$  for the all tests. Two-point crossover was used for two-objective problem and no crossover for three objective problem, due to results which are discussed in Section III. A mutation probability of  $p_m = 0.06$  was used across all the settings and all the test instances. Experiments were conducted using Java 2, on a PC with an Intel Pentium D CPU 3.4 GHz and 2 GB RAM.

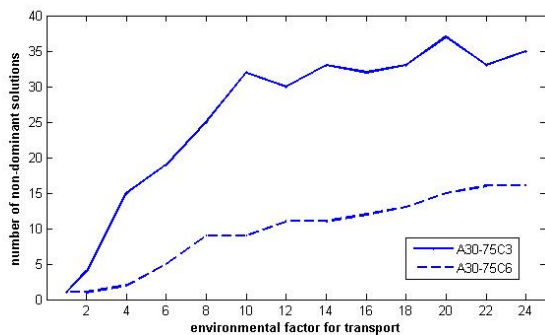


Fig. 2. Total number of non-dominant solutions for different environmental factors from transport for two-objective problem (cost-impact).

## V. RESULTS

In this section we shall attempt to answer the questions posed in the previous section. Consider a situation in which  $W_T = W_F = 1$ . This corresponds to identical objectives for environmental impact and economic cost. For this special case, there is only one global optimum for the economic cost versus environmental impact model, because the problem reduces to a single objective. On the other hand, as the value of  $W_T$  is allowed to increase, one would expect to obtain sets of non-dominant solutions in which high transport impact favours higher numbers of open depots than are cost effective when considered from the point of view of economic cost.

Figure 2 shows how the number of non-dominated solutions obtained by NSGA-II changes as the environmental

factor increases from 1 to 24. This diagram illustrates the situation for two of our instances (A30-75C3 and A30-75C6). However, the pattern is similar for the other 10 instances. We can see that when  $W_T = 1$ , there is a single solution, as expected. As  $W_T$  increases, however, so does the size of the non-dominated set. In the case of A30-75C6, the size of the solution set stabilizes at about 15, while A30-75C3 settles at about 35. The maximum size of the approximate Pareto set is, of course, capped at 40, which is the population size. The curves represent single runs for each  $W_T$  setting (due to time constraints); hence, their lack of smoothness. The solutions depend on the scale - more solutions are found as the environmental impact from transport increases (capped by the population size). Visual representation of the approximate Pareto fronts can be seen in Figure 3 and Figure 4 where a transport factor of 6 was used in the former and a transport factor of 16 in the latter case. As expected, we observe that more depots need to be opened to mitigate the environmental impact than is desirable from the point of view of economic cost. For example, in Figure 3 ( $W_T = 6$ ) the extreme solutions require 2 depots for minimizing cost, and 5 depots for minimizing the environmental impact. In Figure 4 ( $W_T = 16$ ) even more depots (8) are required to mitigate the environmental cost of transport.

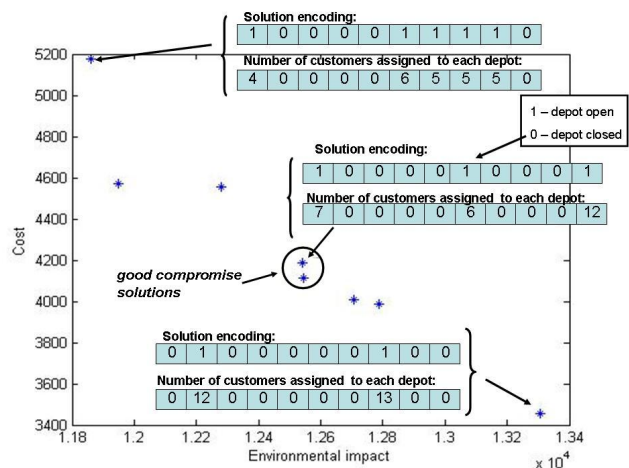


Fig. 3. Instance A10-25C3 with environmental impact factor from depot of 1 and impact factor from transport of 6

Now we are going take a more detailed look at how increases in the environmental factor from transport impacts on the required number of the open depots, for the two-objective and three-objective problems. For each of the 12 settings for  $W_T$  we will examine the extreme solution that minimizes environmental impact at the expense of economic cost (top left of Figures 3 and 4). Figure 6 shows how the number of depots increase with increasing  $W_T$  for two-objective problem, and Figure 7 shows similar findings for the three-objective problem.

Now, to answer the three questions posed in Section IV.

- 1) We do obtain a reasonable trade-off front with a range of solutions, indicating that this approach is worth

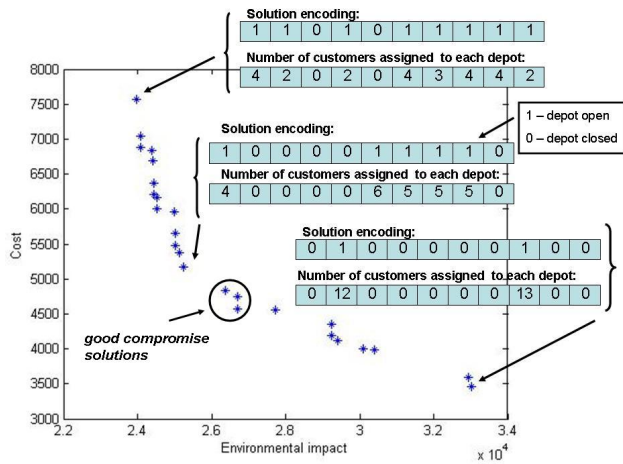


Fig. 4. Instance A10-25C3 with environmental impact factor from depot of 1 and impact factor from transport of 16

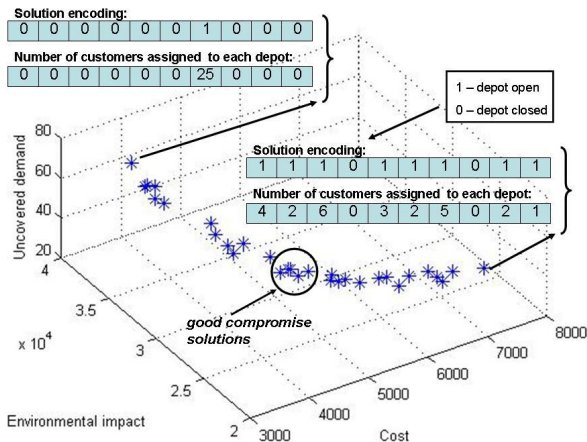


Fig. 5. Instance A10-25C3 with environmental impact factor from depot of 1 and impact factor from transport of 16

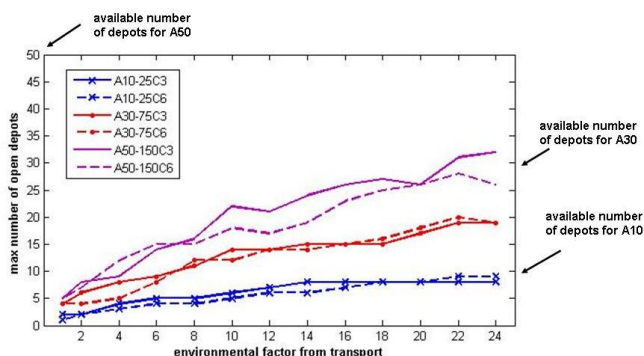


Fig. 6. Total maximum number of depots open depending on environmental impact factor from transport for two-objective problem

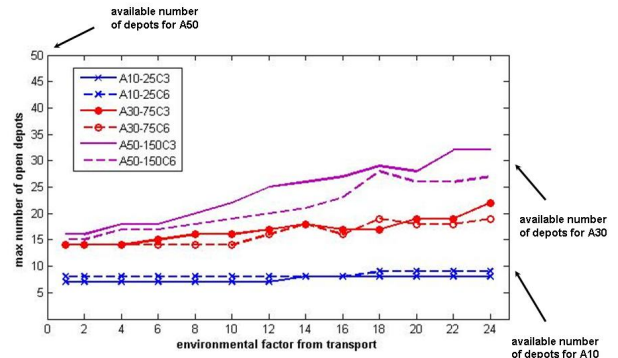


Fig. 7. Total maximum number of depots open depending on environmental impact factor from transport for three-objective problem

pursuing further, until such time that environmental cost is fully absorbed into the economic costs incurred by the stakeholders.

- 2) As the environmental impact of transport increases disproportionately with the cost of operating depots, the environmentally friendly solution will require more open depots than is cost effective from an economic point of view.
- 3) We can spot good compromise solutions, for example as indicated in Figures 3, 4, and 5. We can select solutions with relatively low environmental impact, just before the curve steepens towards very high economic costs. At this stage there are only very small environmental gains to be made at very high economic cost.

## VI. CONCLUSION AND DISCUSSION

This paper describes a multi-objective uncapacitated facility location problem (MOUFLP) with an environmental objective in the context of green logistics design. The model includes traditional objectives, such as minimizing cost and improving customer service level (minimizing uncovered demand) and environmental objectives, such as minimizing the environmental impact from transportation and depots. We apply a multi-objective evolutionary algorithm, NSGA-II, to the MOUFLP to assess the impact of the environmental factor for a range of “what if?” situations, in which we assume that the environmental impact of transport is not truly reflected in the economic costs of running a fleet of vehicles. The analysis was performed on two different settings: a two-objective model (min cost - min environmental impact) and a three-objective model (min cost, min environmental impact and min uncovered demand). The investigation also included the evaluation of the impact of the different scenarios on the number of open depots.

From the two and three objective studies we conclude that it may be desirable to open more depots than may be optimal from a cost only perspective, in order to reduce the

environmental impact of transport. This is not a surprising observation, but our studies indicate that an evolutionary algorithm is a useful way to present trade-off solutions to a human decision maker. It is possible to spot good compromise solutions in this way.

Future plans include extending our exploratory study to a capacitated FLP and a more realistic model. In this model, the environmental impact from transportation and depot management will be extracted using relevant carbon footprint methodology.

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