A Hierarchical Evolutionary Approach to Multi-Objective Optimization

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Introduction

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- Based on the *SEAMO* algorithm (a simple evolutionary algorithm for multi-objective optimization)

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- Based on the *SEAMO* algorithm (a simple evolutionary algorithm for multi-objective optimization)
- A better spread of solutions are obtained if subpopulations of various sizes are used
- Three alternative hierarchical models are tried and the results compared

Why Subpopulations?

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Premature convergence is a serious problem with EAs, and is encountered with single and multi-objective problems alike.

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- They are particularly simple to implement
- No complex global calculations are required for fitness or dominance

Test problems

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- Multiple knapsack problems (MKPs)

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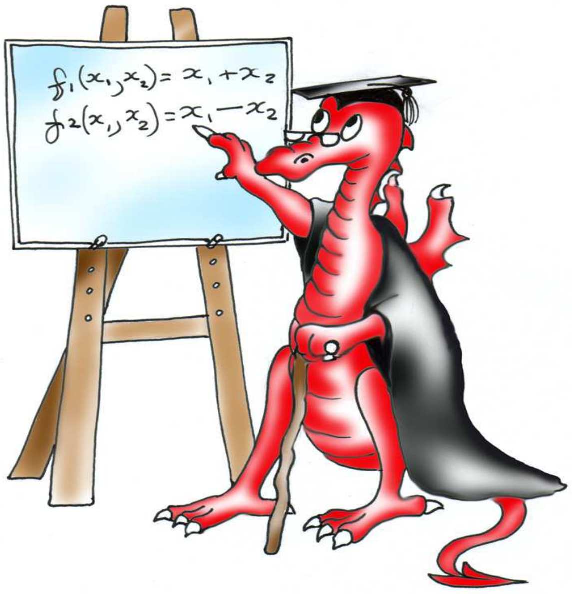
- Continuous functions, SPH-2, ZDT6, QV and KUR



The SEAMO Framework

Procedure *SEAMO*
Begin
Generate N random individuals $\{N$ is the population size$\}$
Evaluate the objective vector for each population member and store it
Repeat
For each member of the population
This individual becomes the first parent
Select a second parent at random
Apply crossover to produce single offspring
Apply a single mutation to the offspring
Evaluate the objective vector produced by the offspring
if offspring qualifies
Then the offspring replaces a member of the population
else it dies
Endfor
Until stopping condition satisfied
Print all non-dominated solutions in the final population
End

Replacement Strategy for SEAMO2

1. **if** offspring harbors a new best-so-far Pareto component
   (a) it replaces a parent, if possible
   (b) **else** it replaces another individual at random
2. **else if** offspring dominates either parent it replaces it
3. **else if** offspring is neither dominated by nor dominates either parent it replaces another individual that it dominates at random
4. **otherwise** it dies

Note: phenotypic duplicates are deleted

Representation for the MKP

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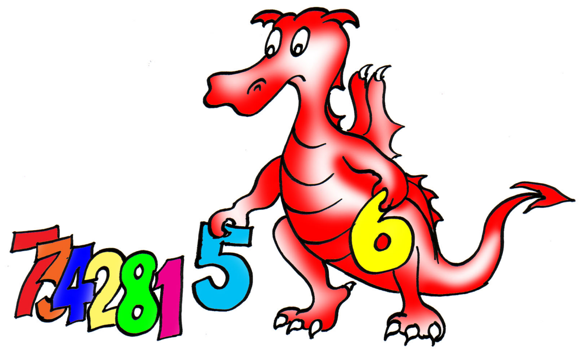

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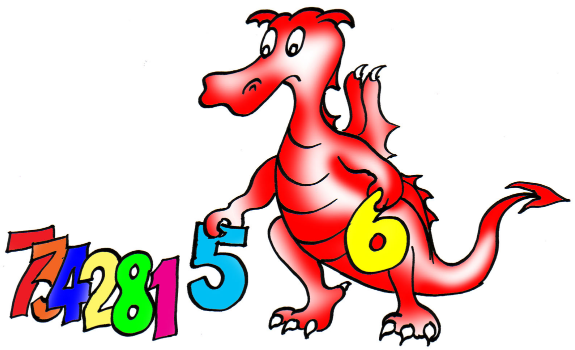

- Order-based representation with a first fit decoder
- Cycle Crossover (CX)
- A simple mutation operator swaps two arbitrarily selected objects within a single permutation list

The Continuous Functions

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- Solutions are coded as real vectors of length 100
- One-point crossover
- A non-uniform mutation
- Deletion of duplicates: component objective functions x_i and x'_i of $\mathbf{x}$ and $\mathbf{x}'$, are equal if and only if

  $x_i - \epsilon \leq x'_i \leq x_i + \epsilon$,

  where ϵ is an error term $(0.00001 \times x_i)$

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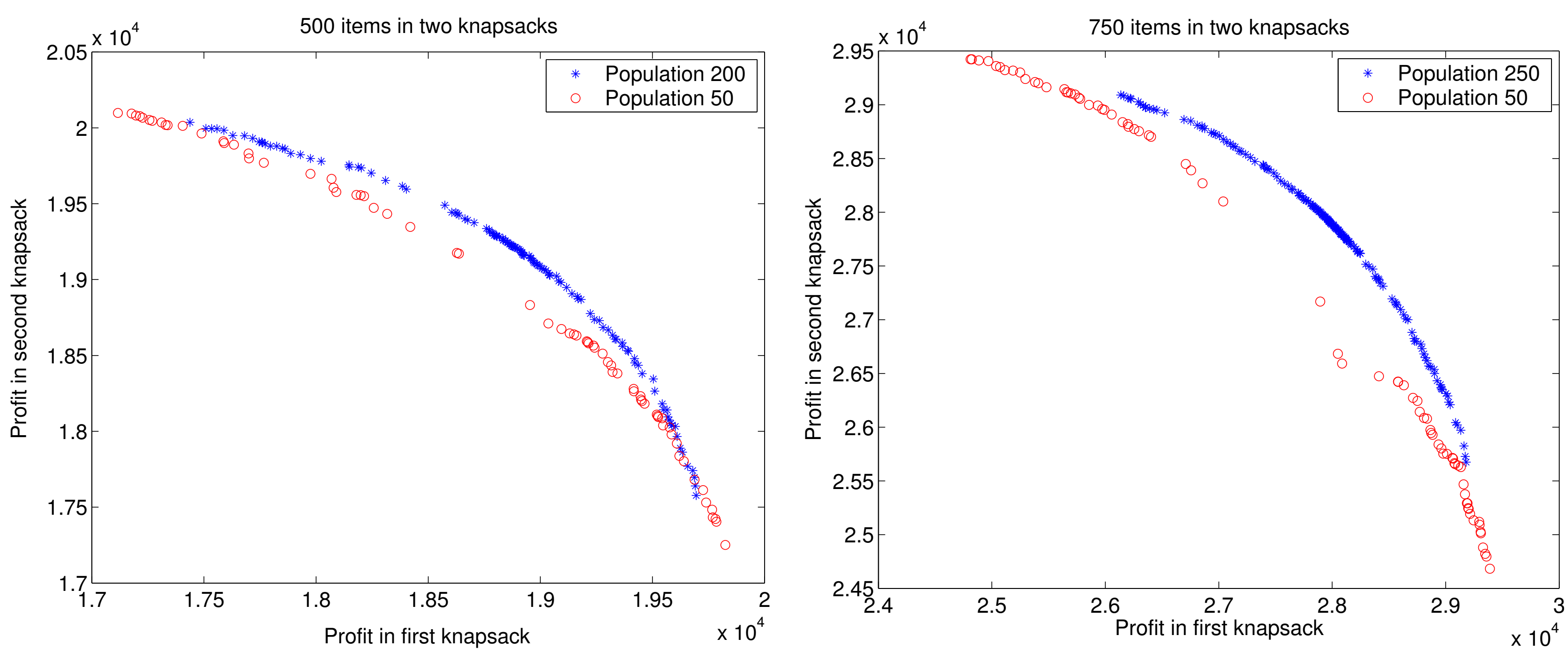
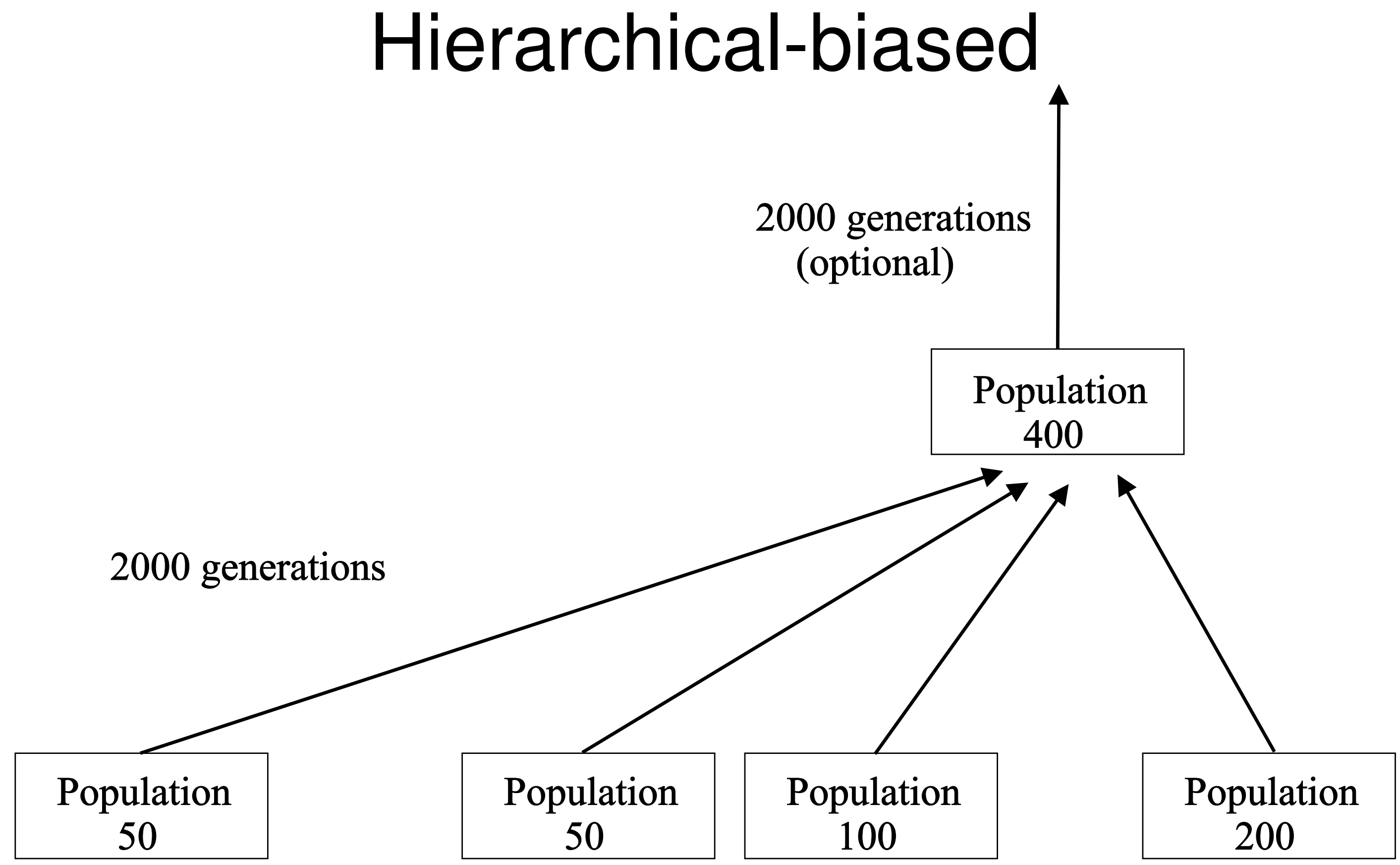
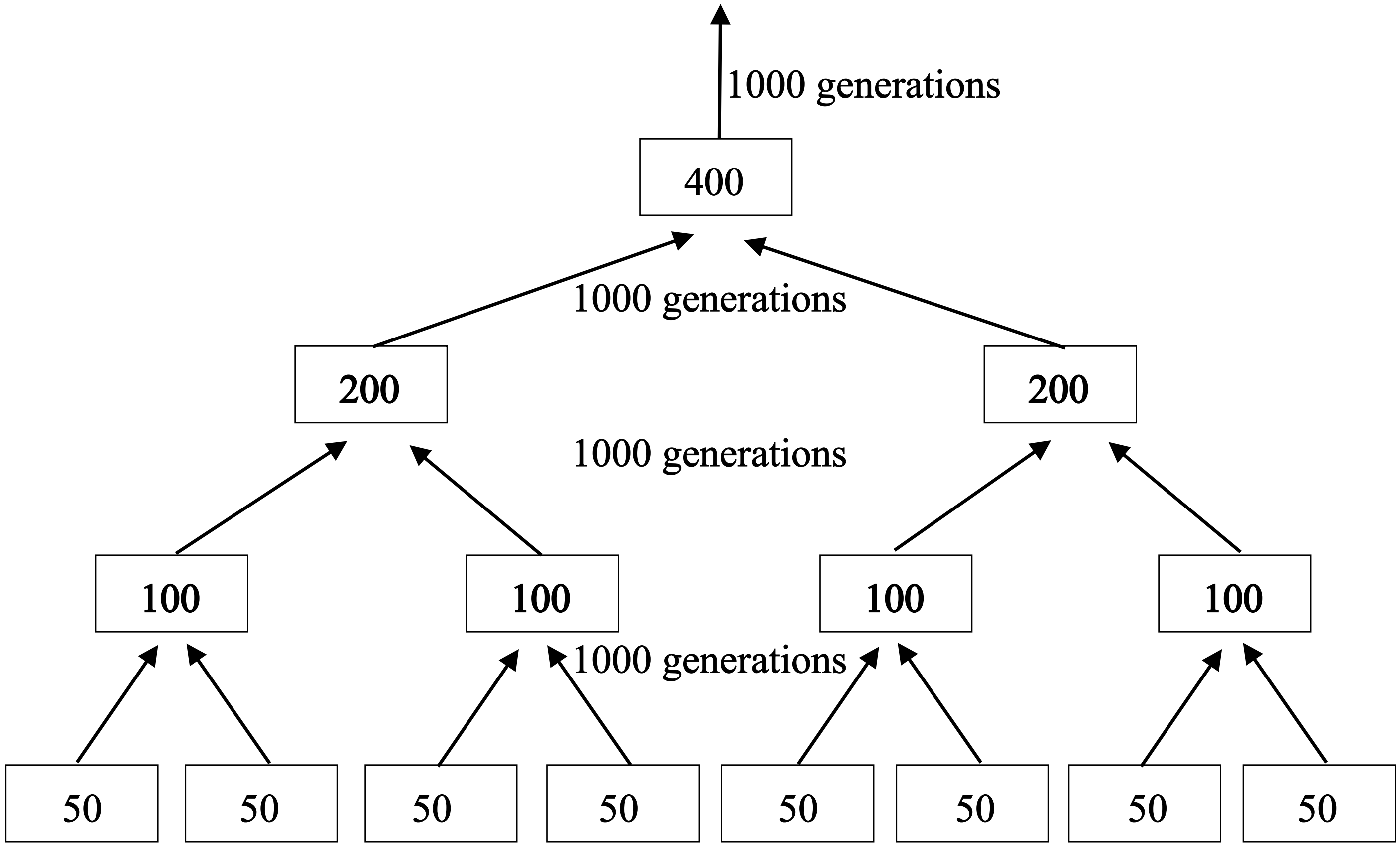


The Hierarchical Algorithms



The Hierarchical Algorithms

Hierarchical-balanced



Hierarchical Biased Algorithms

Procedure *Hierarchical-biased* ($population$)

Begin

if ($populationsize > threshold$)

split $population$ into $leftpop$ and $rightpop$

Hierarchical-biased ($leftpop$)

Run evolutionary algorithm on $rightpop$

else

Run evolutionary algorithm on (unsplit) $population$

End

The Hierarchical Balanced Algorithm

Procedure *Hierarchical-balanced* ($population$)

Begin

if ($populationsize > threshold$)

split population into $leftpop$ and $rightpop$

Hierarchical-balanced ($leftpop$)

Hierarchical-balanced ($rightpop$)

Recombine $leftpop$ and $rightpop$ into $population$

Run evolutionary algorithm on $population$

else

Run evolutionary algorithm on (unsplit) $population$

End

Hierarchical-Biased Algorithms

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- *hierarchical-biased-2layer algorithm* (HBI2)

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- *hierarchical-biased-2layer algorithm* (HBI2)
- *hierarchical-biased-flat algorithm* (HBIF)

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- Experiments are then extended to some continuous functions
- 30 replicate runs carried out for each set of experiments, and all algorithms use the same total population size and number of generations

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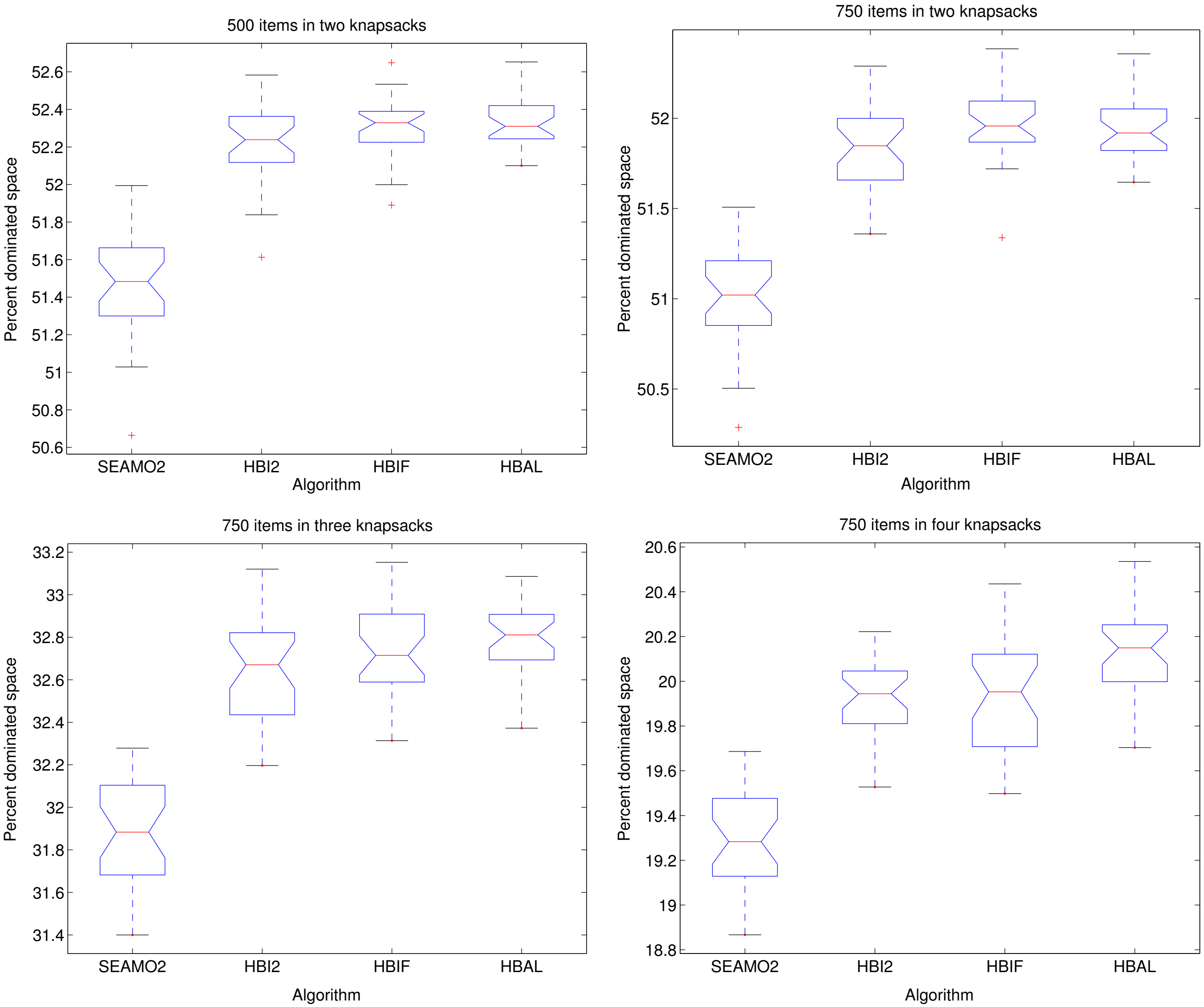
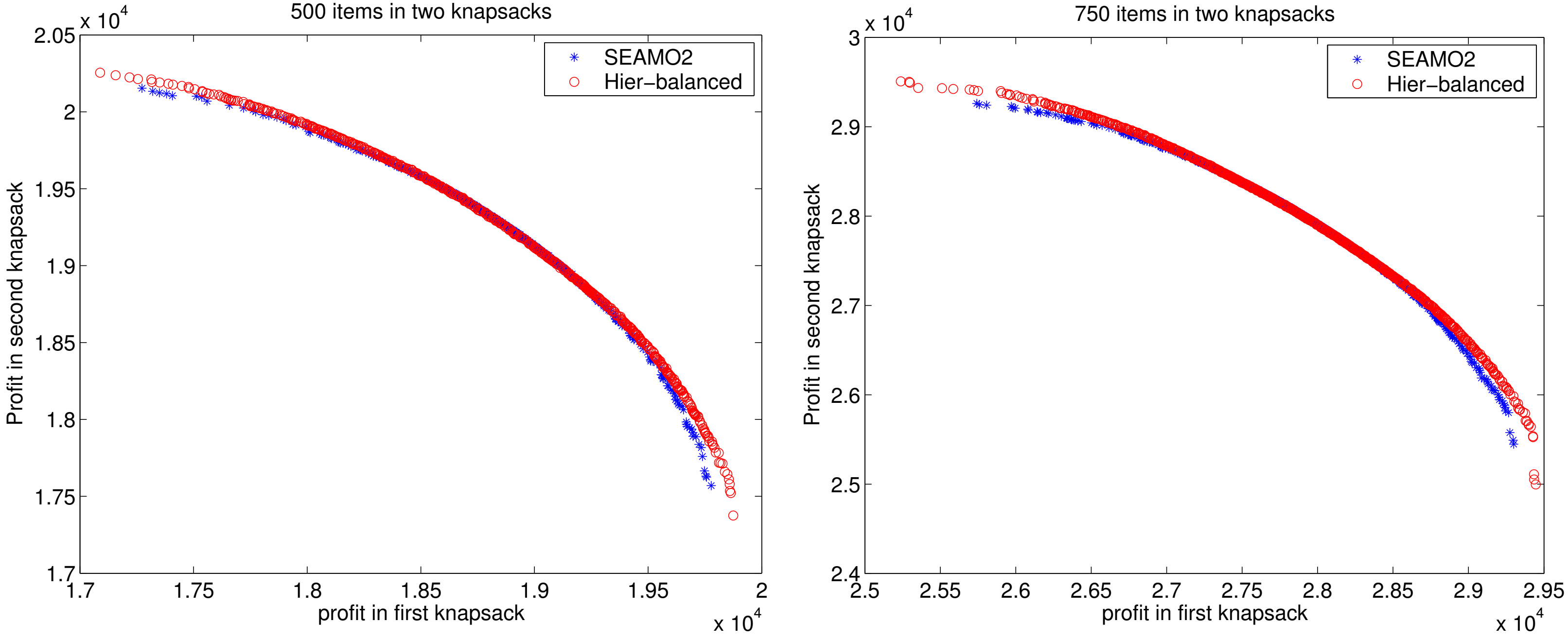
- The hierarchical algorithms are compared only with SEAMO2, and not with any other MOEAs
- SEAMO2 has demonstrated its strength in relation to other EAs elsewhere in a forthcoming GECCO 2004 paper

Results for the MKP, Dominate space, $\mathcal{S}$



Results for Multiple Knapsack Problems



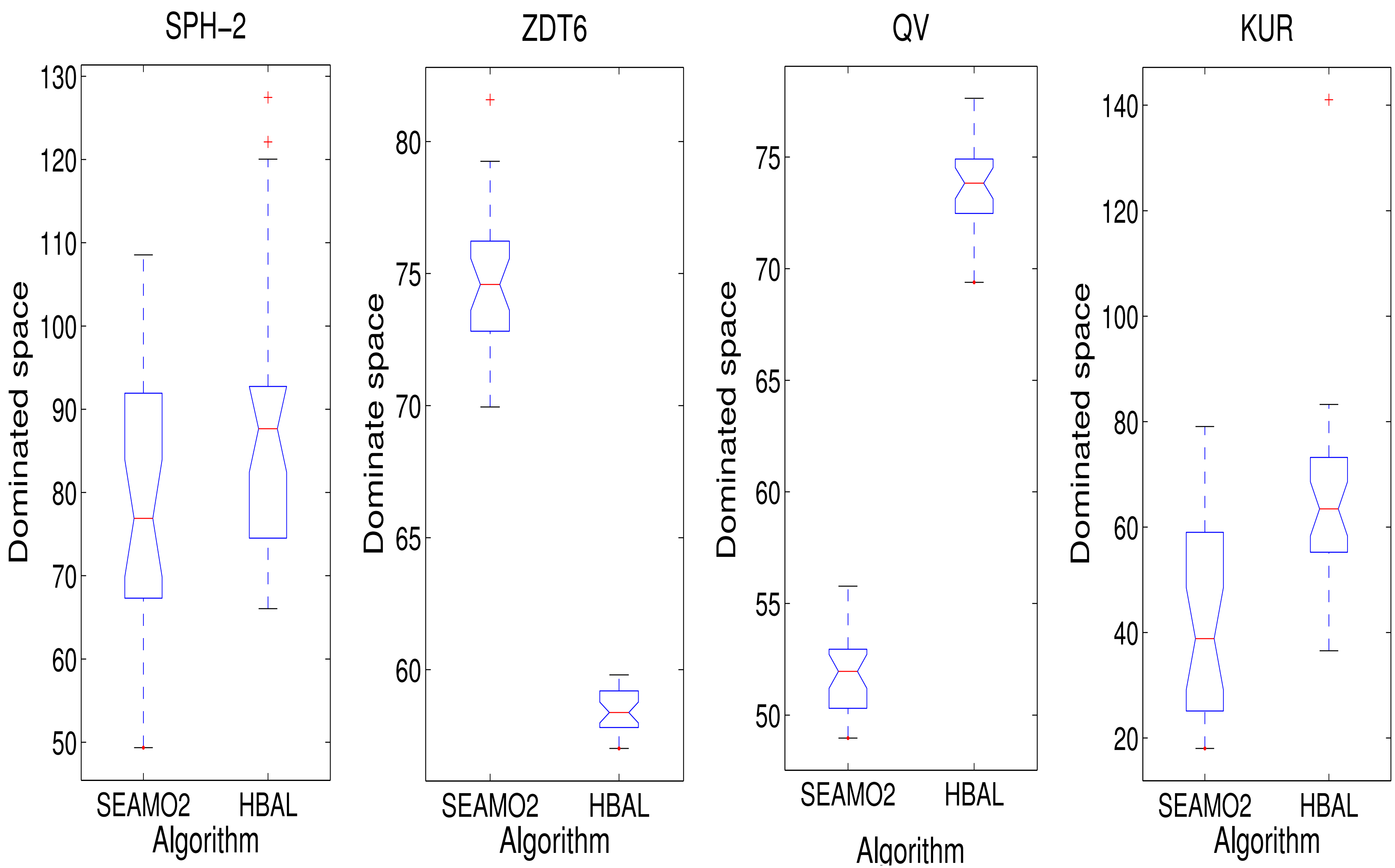
Comparing the performance of *SEAMO2* with

Hierarchical-balanced

Average Coverage, $(A \succeq B)$, on the MKP

Algorithm		Test problems			
A	B	kn500.2	kn750.2	kn750.3	kn750.4
SEAMO2	HBI2	37.6	50.1	46.4	46.9
	HBIF	64.9	70.8	61.4	60.1
	HBAL	22.2	32.8	19.1	21.9
HBI2	SEAMO2	25.4	15.0	5.2	4.4
	HBIF	75.4	68.1	50.1	37.2
	HBAL	20.0	14.8	4.0	6.8
HBIF	SEAMO2	5.8	2.8	0.8	1.1
	HBI2	7,7	14.9	7.3	4.9
	HBAL	5.4	6.1	1.5	2.3
HBAL	SEAMO2	28.9	21.8	12.8	6.4
	HBI2	54.7	72.0	44.4	25.2
	HBIF	72.4	77.1	60.6	50.8

Continuous Function Results, $\mathcal{S}$



Continuous Functions (cont)

Coverage $(A \succeq B)$					
Algorithm		Test problems			
A	B	SPH-2	ZDT6	QV	KUR
SEAMO2	HBAL	4.4	98.9	20.8	10.1
HBAL	SEAMO2	5.4	0	21.6	66.2

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- Better solutions are achieved using large populations and long running times
- Given large computational resources, how do we make best use of them?
- Do we use large single populations or utilize subpopulations?

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- Incorporating runs on small and large populations
- Improving the range of solutions, while maintaining their quality
- The hierarchical balanced algorithm performed best

Future Work

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- Focus subpopulations on different regions of the Pareto space

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- Try ternary, quadtree etc structures for the hierarchical balanced algorithm
- Implement a massively parallel version of SEAMO