Comparing Two Unique Extension Semantics for Formal Argumentation: Ideal and Eager

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Abstract

In formal argumentation, grounded semantics is well known for yielding exactly one unique extension. Since grounded semantics has a very sceptical nature, one can ask the question whether it is possible to define a unique extension semantics that is more credulous. Recent work of Dung, Mancarella and Toni proposes what they call *ideal semantics*, which is a unique extension semantics that is more credulous than grounded semantics. In the current paper, we define a unique extension semantics called *eager semantics* that is even more credulous than ideal semantics. We then examine how this semantics relates to the existing argumentation semantics proposed by Dung and others.

1 Introduction

Formal argumentation has been gaining popularity over the past few years as a human-oriented formalism for non-monotonic entailment [13, 8, 4] and agent-interaction [1, 14]. One issue that has received quite some attention is that of argument semantics. The central question here is, given a set of arguments (some of which defeat others), which possible subsets of arguments can be seen as acceptable.

Traditional approaches to the issue of argument semantics include grounded, stable and preferred semantics [9]. More novel approaches are, for instance, robust semantics [12] and CF2 semantics [2], both of which can, however, yield sets of arguments that are not admissible in the sense of [9].[1] On the other hand, novel approaches like semi-stable semantics [6] do yield extensions that are under any circumstances admissible. In general, the concept of admissibility is a relevant one, not only because it is quite reasonable for a set of accepted arguments to be able to defend itself, but also for technical reasons that are outside of the scope of the current paper.

One of the phenomena in formal argumentation, as well as in nonmonotonic inference in general, is the possibility of a semantics yielding more than one extension (set of accepted arguments). This is often related to an unresolvable dilemma in the available information. Often, the existence of more than one possible extension is dealt with by using a sceptical approach, by taking the intersection of the different extensions. Nevertheless, by taking this intersection, one can lose some of the properties of the individual extensions. For instance, as is explained in Section 3, one can lose the property of admissibility.

For this reason, it can be advantageous to have a semantics that always yields exactly one extension, and that this extension is admissible. The most well-known example of such a unique

[1] Recall that admissibility means that a set of arguments is internally conflict-free and is able to defend itself. That is, a set of arguments $\mathcal{A}rgs$ is admissible iff there exist no $A, B \in \mathcal{A}rgs$ such that A defeats B, and for every D that defeats some argument $C \in \mathcal{A}rgs$, there exists an $E \in \mathcal{A}rgs$ that defeats D.

extension admissible semantics is grounded semantics [9]. A common objection against grounded semantics, however, is that it is supposed to be too sceptical. In order to address this disadvantage, Dung, Mancarella and Toni have recently come up with what they call *ideal semantics* [10], based on earlier work regarding logic programming and assumption based argumentation [11]. The basic idea of ideal semantics will be briefly discussed in Section 3. Then, in Section 4, we introduce the notion of *eager semantics*. As is shown in Section 5, the eager extension is a superset of the ideal extension. Also in Section 5 it is examined how the new as well as existing semantics relate to each other. Some practical considerations regarding the new semantics are then discussed in Section 6.

2 Preliminaries

In this paper we follow the argumentation approach of Dung [9]. For simplicity, we assume the argumentation framework to be finite.

Definition 1 (argumentation framework)**.** *An argumentation framework is a pair* (Ar, def) *where* Ar *is a finite set of arguments and* $def \subseteq Ar \times Ar$.

An argumentation framework can be represented as a directed graph in which the arguments are represented as nodes and the defeat relation is represented as arrows. In several examples throughout this paper, we will use this graph representation.

The shorthand notation A^+ and A^- stands for, respectively, the set of arguments defeated by A and the set of arguments that defeat A. We say that an argument A defeats a set of arguments $\mathcal{A}rgs$ iff it defeats at least one argument in $\mathcal{A}rgs$. We say that a set of arguments $\mathcal{A}rgs$ defeats an argument A iff at least one argument in $\mathcal{A}rgs$ defeats A. We say that a set of arguments $\mathcal{A}rgs_1$ defeats a set of arguments $\mathcal{A}rgs_2$ iff at least one argument in $\mathcal{A}rgs_1$ defeats at least one argument in $\mathcal{A}rgs_2$. In the definition below, $F(\mathcal{A}rgs)$ stands for the set of arguments that are acceptable in the sense of [9].

Definition 2 (defense / conflict-free)**.** *Let* $A \in Ar$ *and* $\mathcal{A}rgs \subseteq Ar$.
We define A^+ *as* $\{B \mid A\ def\ B\}$ *and* $\mathcal{A}rgs^+$ *as* $\{B \mid A\ def\ B$ *for some* $A \in \mathcal{A}rgs\}$.
We define A^- *as* $\{B \mid B\ def\ A\}$ *and* $\mathcal{A}rgs^-$ *as* $\{B \mid B\ def\ A$ *for some* $A \in \mathcal{A}rgs\}$.
$\mathcal{A}rgs$ *is* conflict-free *iff* $\mathcal{A}rgs \cap \mathcal{A}rgs^+ = \emptyset$.
$\mathcal{A}rgs$ defends *an argument* A *iff* $A^- \subseteq \mathcal{A}rgs^+$.
We define the function $F : 2^{Ar} \to 2^{Ar}$ *as*
$F(\mathcal{A}rgs) = \{A \mid A$ *is defended by* $\mathcal{A}rgs\}$.

In the definition below, definitions of grounded, preferred and stable semantics are described in terms of complete semantics, which has the advantage of making the proofs in the remainder of this paper more straightforward. These descriptions are not literally the same as the ones provided by Dung [9], but as was first stated in [5] they are in fact equivalent to Dung's original versions of grounded, preferred and stable semantics.

Definition 3 (acceptability semantics)**.** *Let* $\mathcal{A}rgs$ *be a conflict-free set of arguments.*

- $\mathcal{A}rgs$ *is* admissible *iff* $\mathcal{A}rgs \subseteq F(\mathcal{A}rgs)$.
- $\mathcal{A}rgs$ *is a* complete *extension iff* $\mathcal{A}rgs = F(\mathcal{A}rgs)$.
- $\mathcal{A}rgs$ *is a* grounded *extension iff* $\mathcal{A}rgs$ *is the minimal (w.r.t. set-inclusion) complete extension.*
- $\mathcal{A}rgs$ *is a* preferred *extension iff* $\mathcal{A}rgs$ *is a maximal (w.r.t. set-inclusion) complete extension.*
- $\mathcal{A}rgs$ *is a* semi-stable *extension iff* $\mathcal{A}rgs$ *is a complete extension where* $\mathcal{A}rgs \cup \mathcal{A}rgs^+$ *is maximal (w.r.t. set-inclusion).*
- $\mathcal{A}rgs$ *is a* stable *extension iff* $\mathcal{A}rgs$ *is a complete extension that defeats every argument in* $Ar \backslash \mathcal{A}rgs$.

The following lemma is used in the proofs of the next section.

Lemma 1. *Let $\mathcal{A}rgs_1$ and $\mathcal{A}rgs_2$ be two admissible sets of argumentation framework (Ar, def). If $\mathcal{A}rgs_1$ and $\mathcal{A}rgs_2$ do not defeat each other then $\mathcal{A}rgs_1 \cup \mathcal{A}rgs_2$ is again an admissible set.*

Proof. We need to prove two things:

1. $\mathcal{A}rgs_1 \cup \mathcal{A}rgs_2$ is conflict-free. This follows from the fact that $\mathcal{A}rgs_1$ and $\mathcal{A}rgs_2$ are individually conflict-free and do not defeat each other.
2. $\mathcal{A}rgs_1 \cup \mathcal{A}rgs_2$ defends all its elements. This follows from the fact that $\mathcal{A}rgs_1$ and $\mathcal{A}rgs_2$ respectively defend all its elements.

□

3 Ideal Semantics

Ideal semantics was proposed by Dung, Mancarella and Toni [10] based on earlier work regarding logic programming and assumption based argumentation [11].[2] The idea of ideal semantics is to be slightly more sceptical than just taking the intersection of all preferred extensions (sceptical preferred). The point is that this intersection might not be an admissible set itself. Consider the example of Figure 1.

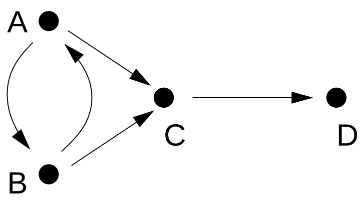

Figure 1: Sceptical preferred yields $\{D\}$, which is not admissible.

In Figure 1, there are two preferred extensions: $\{A, D\}$ and $\{B, D\}$. Their intersection (sceptical preferred) is $\{D\}$, which is itself not admissible. However, it turns out that there always exists a unique largest admissible subset of this intersection, as is shown in the following theorem.

Theorem 1. *Let (Ar, def) be an argumentation framework. There exists exactly one maximal (w.r.t. set-inclusion) admissible set $\mathcal{A}rgs$ that is a subset of each preferred extension.*

Proof. As the existence of a maximal set $\mathcal{A}rgs$ is guaranteed, we only need to prove uniqueness. Suppose there are two different maximal admissible sets $\mathcal{A}rgs_1$ and $\mathcal{A}rgs_2$ that both subsets of each preferred extension. The fact that $\mathcal{A}rgs_1$ and $\mathcal{A}rgs_2$ are in each preferred extension (and the fact that there exists at least one preferred extension) means that $\mathcal{A}rgs_1$ and $\mathcal{A}rgs_2$ do not defeat each other. Since the union of two admissible sets that do not defeat each other is itself again an admissible set (Lemma 1) it holds that $\mathcal{A}rgs_3 = \mathcal{A}rgs_2 \cup \mathcal{A}rgs_1$ is an admissible set that is a subset of each preferred extension. But as $\mathcal{A}rgs_1 \neq \emptyset$ and $\mathcal{A}rgs_2 \neq \emptyset$ it follows that $\mathcal{A}rgs_3 \supsetneqq \mathcal{A}rgs_1$ and $\mathcal{A}rgs_3 \supsetneqq \mathcal{A}rgs_2$. Therefore, $\mathcal{A}rgs_1$ and $\mathcal{A}rgs_2$ are not maximal. Contradiction. □

[2]The technical report of Dung, Mancarella and Toni [10] became available during the preparation of the current paper. We have, however, to some extent stuck to our own definitions and proofs in this section, since this makes it easier to compare, for instance, ideal semantics to eager semantics, to be defined in the next section. It should be mentioned, however, that the notion of ideal semantics discussed in the current paper is the same as that in [10]. One small difference is that what we call an *ideal extension* is called a *maximal ideal set* in [10], where a set of arguments is ideal iff it is admissible and a subset of each preferred extension.

The fact that the maximal admissible set that is a subset of each preferred extension is unique, allows one to refer to it, like is done in the following definition.

Definition 4. *The* ideal extension *is the greatest (w.r.t. set-inclusion) admissible set that is a subset of each preferred extension.*

To see how ideal semantics works, consider figure 2. Here, the grounded extension is empty, and there exists just one preferred extension: $\{A\}$. The ideal extension is therefore $\{A\}$.

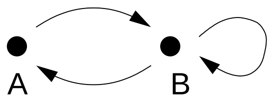

Figure 2: Grounded extension: $\emptyset$; ideal extension: $\{A\}$; eager extension: $\{A\}$.

It can be observed that the ideal extension is itself a complete extension.

Theorem 2. *Let (Ar, def) be an argumentation framework and $\mathcal{A}rgs$ be its ideal extension. It holds that $\mathcal{A}rgs$ is a complete extension.*

Proof. We have to prove that $\mathcal{A}rgs = F(\mathcal{A}rgs)$.
$\mathcal{A}rgs \subseteq F(\mathcal{A}rgs)$: This follows directly from the fact that the ideal extension is an admissible set.
$F(\mathcal{A}rgs) \subseteq \mathcal{A}rgs$: Suppose $A \in F(\mathcal{A}rgs)$. Then A is defended by every preferred extension (this follows from the monotonity of F). As every preferred extension is a complete extension, this means that A is an element of every preferred extension. From this, it follows that $\mathcal{A}rgs \cup \{A\}$ is an admissible set, one that is a subset of each preferred extension. Therefore A must be an element of the ideal extension $\mathcal{A}rgs$. □

4 Eager Semantics

Semi-stable semantics [6] is a recently developed semantics that could be placed between preferred and stable semantics in the sense that every stable extension is also a semi-stable extension, and every semi-stable extension is also a preferred extension. Moreover, it has been proved that if there exists at least one stable extension, then the semi-stable extensions are the same as the stable extensions. The advantage of semi-stable semantics above stable semantics is that, for any finite argumentation framework, a semi-stable extension always exists.

Like the ideal extension is the greatest admissible set that is a subset of each extension, one can define the eager extension as the greatest admissible set that is a subset of each semi-stable extension.

Theorem 3. *Let (Ar, def) be an argumentation framework. There exists exactly one maximal (w.r.t. set-inclusion) admissible set $\mathcal{A}rgs$ that is a subset of each semi-stable extension.*

Proof. Similar to the proof of Theorem 1. □

Definition 5. *The* eager extension *is the greatest (w.r.t. set-inclusion) admissible set that is a subset of each semi-stable extension.*

To see how eager semantics works, consider figure 3. Here, the grounded extension is empty. There exist two preferred extensions: $\{A\}$ and $\{B, D\}$. The intersection of the preferred extensions is empty, from which it follows that the ideal extension is the empty set. There exists just one semi-stable extension: $\{B, D\}$, which implies that the eager extension is $\{B, D\}$.

Theorem 4. *Let (Ar, def) be an argumentation framework and $\mathcal{A}rgs$ be its eager extension. It holds that $\mathcal{A}rgs$ is a complete extension.*

Proof. Similar to the proof of Theorem 1. □

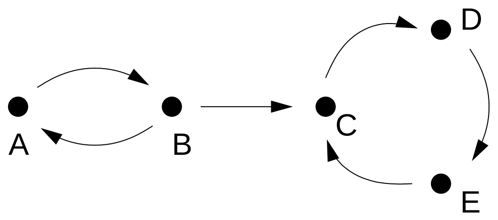

Figure 3: Grounded extension: $\emptyset$; ideal extension: $\emptyset$; eager extension: $\{B, D\}$.

5 Comparing Grounded, Ideal and Eager Semantics

It is interesting to examine how grounded, ideal and eager semantics relate to each other. It turns out that the grounded extension is a subset of the ideal extension, and the ideal extension is itself a subset of the eager extension.

Theorem 5. *Let (Ar, def) be an argumentation framework. Let $\mathcal{A}rgs_{GR}$ be the grounded extension, $\mathcal{A}rgs_{ideal}$ the ideal-extension and $\mathcal{A}rgs_{eager}$ the eager extension. It holds that $\mathcal{A}rgs_{GR} \subseteq \mathcal{A}rgs_{ideal} \subseteq \mathcal{A}rgs_{eager}$.*

Proof. We need to prove two things:

1. $\mathcal{A}rgs_{GR} \subseteq \mathcal{A}rgs_{ideal}$. In [9] it is proved that the grounded extension is a subset of every complete extension. From the fact that $\mathcal{A}rgs_{ideal}$ is a complete extension (Theorem 2) it then follows that $\mathcal{A}rgs_{GR} \subseteq \mathcal{A}rgs_{ideal}$.
2. $\mathcal{A}rgs_{ideal} \subseteq \mathcal{A}rgs_{eager}$. In [5] it is proved that each semi-stable extension is also a preferred extension. This implies that the intersection of all semi-stable extensions (sceptical semi-stable) is a superset of the intersection of all preferred extensions (sceptical preferred). From this, it follows that the biggest admissible subset of the intersection of all semi-stable extensions (the eager extension) is a superset of the intersection of all preferred extensions (the ideal extension).

□

As is stated in Theorem 5, (i) the grounded extension is a subset of the ideal extension, and (ii) the ideal extension is a subset of the eager extension. Moreover, by Definition 4, we also have that (iii) the ideal extension is a subset of each preferred extension. Definition 5 then states that (iv) the eager extension is a subset of each semi-stable extension. From the fact that each stable extension is also a semi-stable extension, it then follows that (v) the eager extension is a subset of every stable extension. From [9] it also follows that (vi) the grounded extension is a subset of every complete extension. This yields the overall picture of Figure 4.

It can also be observed that [7, 6, 9] (1) every stable extension is a semi-stable extension, (2) every semi-stable extension is a preferred extension, (3) every preferred extension is a complete extension and (4) the grounded extension is a complete extension. Moreover, we also have that (5) the ideal extension is a complete extension and (6) the eager extension is a complete extension. This yields the overall picture of Figure 5.

6 Discussion

In this paper, we have introduced the concept of eager semantics, and shown how it relates to various existing argumentation semantics. In this way, we aim to contribute to the task of mapping out the space of possible admissibility based semantics, as is done in Figure 4 and 5.

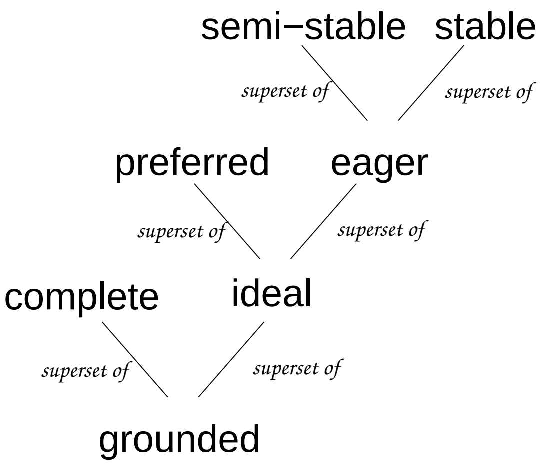

Figure 4: Overview of admissibility based semantics (subset)

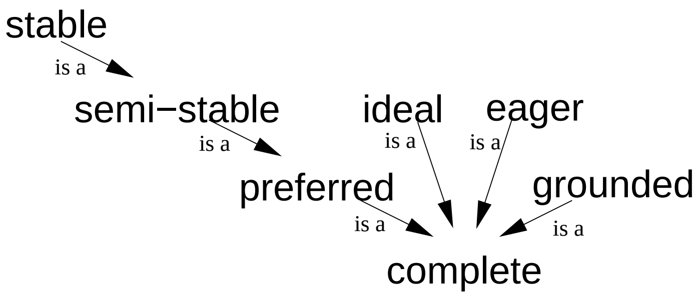

Figure 5: Overview of admissibility based semantics (specialization)

One important issue regarding argumentation semantics is that of proof procedures and computational complexity. As for ideal semantics, proof procedures are given in [10]. Basically, the idea is to determine whether an argument is in the ideal extension by trying to construct an admissible set that contains this argument and is not defeated by another admissible set. For eager semantics, the situation is slightly more difficult. Although proof procedures have been stated for semi-stable semantics [3], these are yet to be adjusted to apply to eager semantics. Since semi-stable semantics coincides with stable semantics for cases where stable extensions exist, the complexity of semi-stable semantics is equal to the complexity of stable semantics for argumentation frameworks that have stable extensions. Although a full complexity study is still to be done for eager as well as for ideal semantics, it is unlikely for the complexity of eager semantics to be anywhere below the complexity of ideal semantics.

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