Strong admissibility in acyclic argumentation frameworks

Martin Caminada

June 2020

The current technical report uses the definitions of [1] and should be read together with it. It presents a theorem and proof that is used in [1].

Theorem 1. *Let $AF = (Ar, att)$ be an acyclic argumentation framework (so without any directed cycles). Each admissible labelling of AF is also a strongly admissible labelling of AF.*

Proof. The fact that AF is an acyclic directed graph means that it defines a partial order among the arguments: $A \leq B$ iff either $A = B$ of A is an ancestor of B.[1] Let $\mathcal{L}ab_{adm}$ be an admissible labelling of AF and let $\mathcal{MM}_{adm}$ be its associated min-max numbering. Assume, towards a contradiction, that $\mathcal{L}ab_{adm}$ is not strongly admissible. This means there exists an argument that is labelled `in` or `out` by $\mathcal{L}ab_{adm}$ and is numbered ∞ by $\mathcal{MM}_{adm}$. Let $Args$ be the set of arguments that are labelled `in` or `out` by $\mathcal{L}ab_{adm}$ and numbered ∞ by $\mathcal{MM}_{adm}$. Let A be a minimal element (w.r.t. the partial order induced by AF) of $\mathcal{A}rgs$. We distinguish two cases:

1. $\mathcal{L}ab_{adm}(A) = \texttt{in}$
   From $\mathcal{L}ab_{adm}$ being an admissible labelling it follows that $\mathcal{L}ab_{adm}(B) = \texttt{out}$ for each attacker B of A. From the fact that $\mathcal{MM}_{adm}(A) = \infty$ it follows [1][Definition 8, first bullet point] that $\max(\{\mathcal{MM}_{adm}(B) \mid B \text{ attacks } A \text{ and } \mathcal{L}ab_{adm}(B) = \texttt{out}\}) + 1 = \infty$. This implies that there exists at least one `out` labelled attacker B of A with $\mathcal{MM}_{adm}(B) = \infty$. But then A would not have been a minimal element of $\mathcal{A}rgs$. Contradiction.

2. $\mathcal{L}ab_{adm}(B) = \texttt{out}$
   From $\mathcal{L}ab_{adm}$ being an admissible labelling it follows that there is an attacker B of A such that $\mathcal{L}ab_{adm}(B) = \texttt{in}$. From the fact that $\mathcal{MM}_{adm}(A) = \infty$ it follows [1][Definition 8, second bullet point] that $\min(\{\mathcal{MM}_{adm}(B) \mid B \text{ attacks } A \text{ and } \mathcal{L}ab_{adm}(B) = \texttt{in}\}) + 1 = \infty$. This implies that there exists at least one `in` labelled attacker B of A with $\mathcal{MM}_{adm}(B) = \infty$. But then A would not have been a minimal element of $\mathcal{A}rgs$. Contradiction.

□

An acyclic argumentation framework is well-founded in the sense of [2][Definition 29]. It therefore follows that it has a single preferred extension, which is also grounded [2][Theorem 30]. As a preferred extension is a maximal admissible set, and a grounded extension is a (unique) maximal strongly admissible set, Theorem 1 can to some extent be seen as generalising [2][Theorem 30].

References

[1] M.W.A. Caminada and P.E. Dunne. Minimal strong admissibility: a complexity analysis. In *Proceedings of COMMA 2020*. IOS Press, 2020.

[2] P.M. Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77:321–357, 1995.

[1] A is an ancestor of B iff either $(A, B) \in att$, or there exists an argument C such that $(C, B) \in att$ and A is an ancestor of C.